Basics and statics of Particles

Definition:- Engineering mechanics is that branch of science which deals with the behaviour of a body when the body is at rest or in motion.

Engineering mechanics

- 1. Statics (body is at rest)
- 2. Dynamics (body is in motion)
  - (i) Kinematics
  - (ii) Kinetics.

Kinematics:- The study of a body in motion, when the forces which cause the motion are not considered is called κ.

Kinetics:- if the forces are also considered for the body in motion, that branch of science is called kinetics.

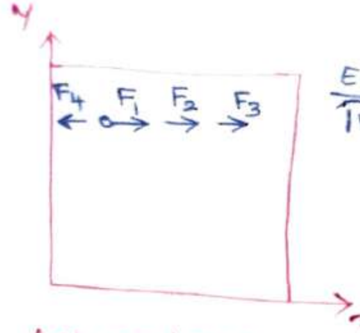
Engineering mechanics

- Mechanics of solids
  - Rigid bodies
    - 1. Statics
    - 2. Dynamics
      - kinetics
      - kinematics.
  - Deformable bodies.
    - 1. Strength of materials
    - 2. Theory of Elasticity
    - 3. Theory of Plasticity.
- Mechanics of fluids.
  - * Ideal fluids
  - * viscous fluids
  - * compressible fluids.

Force System.

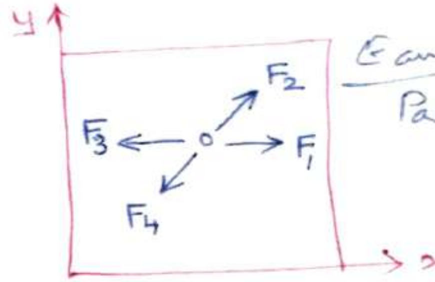
- Coplanar (plane)
  - collinear
  - concurrent
  - Parallel
  - Non-concurrent
  - Nonparallel.
- Non-coplanar (space)
  - concurrent
  - Parallel
  - Non-concurrent
  - Nonparallel

Forces



Example
Tug of war

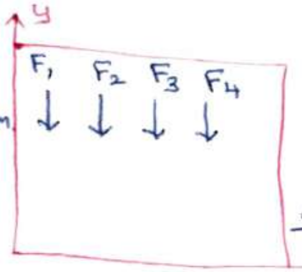
(a) coplanar - collinear



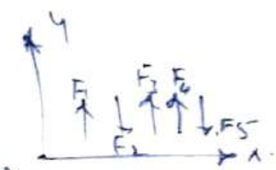
Example.
Pascal's Law

(b) coplanar - concurrent
(same point)

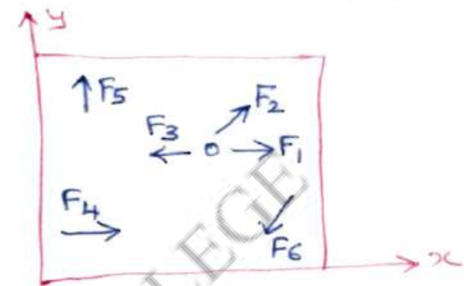
Example.
Concentrated load on a beam
Co-planar like parallel forces



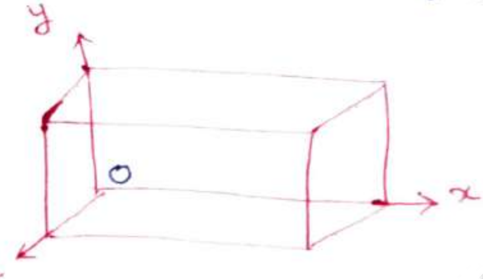
(c) coplanar - parallel



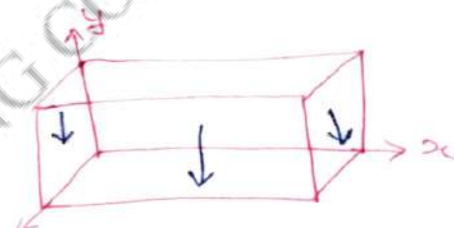
C(ii) Co-planar unlike parallel forces.
Example forces on a truss.



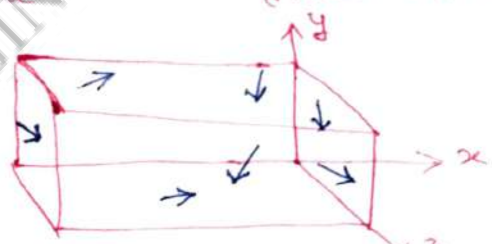
(d) coplanar - non concurrent



(e) Non- coplanar, concurrent



(f) Non coplanar, parallel forces.



(g) Non- coplanar, non-concurrent.
non- parallel.

Characteristics of Force systems:

Sl. No.	Force System	Characteristics.
1.	coplanar forces	Lines of action of all forces lie on the same plane
2.	Non-coplanar forces	Lines of action of all forces act along the same ~~plane~~ plane.
3.	collinear forces	Lines of action of all forces act along the same line.
4.	concurrent forces	Lines of action of all forces pass through a single point.
5.	Parallel forces.	Lines of action of all forces that are parallel to each other.

(1) **Laws of mechanics**

The study of rigid body mechanics is based on the ~~fol~~ 3 laws of mechanics.

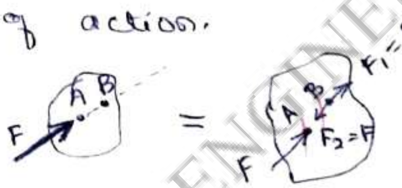
First law: A particle remain in its position (rest or motion), if the resultant force acting on the particle is zero.

Second law: Acceleration of the particle will be proportional to the resultant force and in the same direction, if the resultant force is not zero.

Third law: Action and reaction forces between the interacting bodies are in the same line of action, equal in magnitude but acts in the opp. direction.

(2) **Principle of transmissibility of force:**

If a force acts at any pt. on a rigidbody it may also be considered to act at any other point on its line of action.

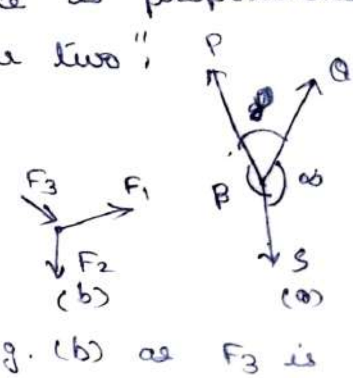


equal & opp
hence cancels

(3) **Lami's theorem:**

It states that "if 3 coplanar forces acting at a pt. be in equilibrium, then each force is proportional to the sine of the angle bet. the other two".

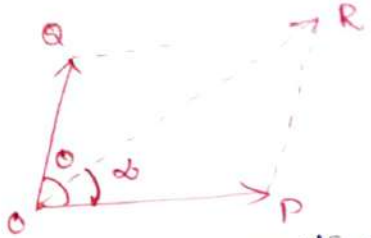
Mathematically, $\dfrac{P}{\sin\alpha} = \dfrac{Q}{\sin\beta} = \dfrac{S}{\sin\gamma}$.



This law is not applicable to fig. (b) as F_3 is acting towards centre. [For Lami's theorem to be applicable, all forces should act away from the centre]

4) Parallelogram law of forces:

It states that " If two forces acting simultaneously at a point be represented in magnitude and direction by two adjacent sides of a parallelogram, then the resultant of these forces is represented in magnitude and direction by the diagonal of that parallelogram originating from that point.



$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

Mathematically $\boxed{R = \sqrt{P^2 + Q^2 + (2PQ\cos\theta)}}$

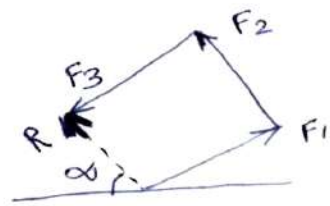
α = direction of resultant force made with eastern direction.

5) a) Triangle law of forces:

It states that " If two forces acting at a pt. are represented by the two sides of a triangle taken in order, then their resultant force is represented by the third side taken in opposite order.

b) Polygon law of forces

It states that " if a number of coplanar concurrent forces are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant force is represented by the closing side of the polygon taken in the opposite order".



6) Scalar and vector Quantities:

Scalar Quantities: The quantities which possess magnitude only are called scalar quantities

eg: Length, area, volume, mass etc.

Vector Quantities: The quantities which possess magnitude as well as direction are called vector quantities.

examples: forces, velocity, acceleration etc.

7 Newton's First law of motion: (Inertia)

Every object will remain at rest or in uniform motion in a straight line unless acted on by an external force.

Suggested Questions / Assignments / Home works / any other

Study the characteristics of forces systems and laws involved.

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N.Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston Jr., David F Mazurek, Philip J Cornwell, SanjeevSanghi,	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press

Any other suggested Materials

Class notes and Handouts

Moment of a force: is defined as the product of magnitude of the force and the perpendicular distance between the moment point and pt. of application of the force.

Resultant of coplanar parallel forces:

Resultant of two parallel forces.

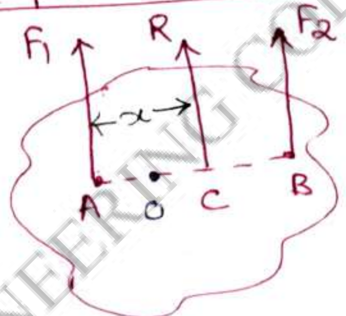
Three cases of two parallel forces are considered.

(i) Two like parallel forces.

(ii) Two unlike parallel forces of unequal magnitude.

(iii) Two unlike parallel forces of equal magnitude

1. Resultant of two like parallel forces:



The resultant force R is given as

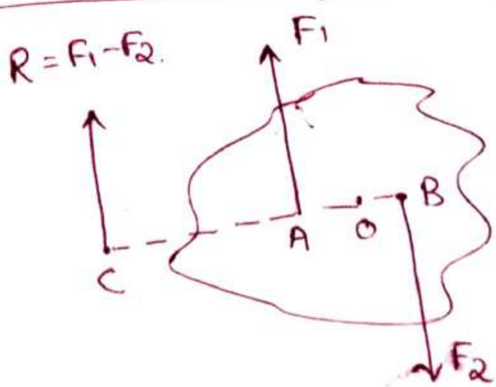
$$\boxed{R = F_1 + F_2.}$$

applying varignon's theorem,
next page
taking moment about A)

$$-(F_2 \times AB) + (F_1 \times 0) - (R \times \overset{x}{AC}) = 0$$

Find out the value for x.

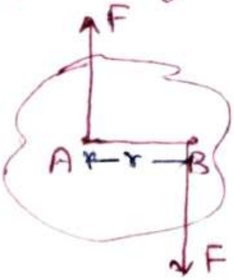
2. Resultant of two unlike parallel forces of unequal magnitude



Resultant $R = F_1 - F_2$.

3. Resultant of two unlike parallel forces of equal magnitude

This is a couple which has a tendency to rotate the body and the $\boxed{\text{resultant is zero}}$.



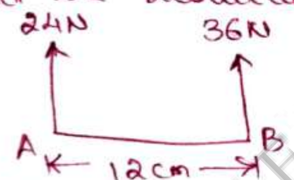
Here the $\boxed{\text{moment, } M = +F \times r}$.

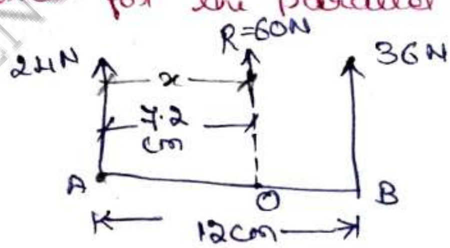
Varignon's theorem: The algebric sum of the moments of any no. of forces about any point in their plane is equal to the moment of their resultant about the same point.

Varignon's theorem is also known as theorem of moments.

$$\left.\begin{array}{l}\text{sum of moment of all}\\ \text{forces about a point}\end{array}\right\} = \begin{array}{l}\text{moment of their resultant}\\ \text{force about the same point.}\end{array}$$

① Find the resultant force for the parallel force system.





$\boxed{R = 24 + 36 = 60N.}$

To locate the resultant, take moment about A.

$R \times x = (A \times 0) - B(12)$. $\quad - 7.2$

$60x = -(12 \times 36)$ $\Rightarrow$ ~~xxxxxxxx~~.

(-ve sign for anticlockwise moment).

$\to R x + 36(12)$ ...

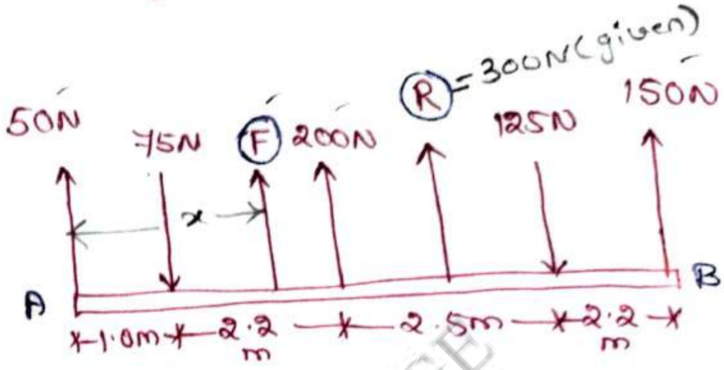
$R \times x =$

$60x = -12 \times 36$

$\boxed{x = 7.2\,m}$

(2) Six parallel forces of magnitude as shown in fig. are acting on a beam AB. If the resultant R = 300N and is acting at a distance of 4.5m from A, find (i) Magnitude of the unknown force F (ii) Distance of F from A.

(i) Magnitude of force F,



$R = (50 + 200 + F + 150) - 75 - 125$

$R = (200 + F)$ N.

Sub. R = 300N in the above eqn.

$\therefore F = 100N$ acting upwards. (+ve. hence acting upwards).

(ii) Distance of F from A. (assume as x)

Taking moment of all the forces about A, [clockwise +ve, anti " -ve]

$(-50 \times 0) + (75 \times 1) + (125 \times 5.7) - (F \times x) - (200 \times 3.2) - (150 \times 7.9) = (R \times 4.5)$

$0 + 75 - 640 + 712.5 - 1185 - 100x = (300 \times 4.5)$

$\boxed{x = 3.125m.}$

The force F acts at 3.125m from pt. A.

Lecture No. 03

UNIT I - STATICS OF PARTICLES

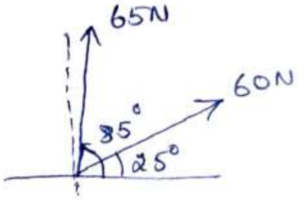
Topic(s) to be covered	statics of particles in two dimensions – Resultant force

	Lecture Outcome (LO) **At the end of this lecture, students will be able to**	**Bloom's Level**
LO1	find resultant force.	K3 apply.

Teaching Learning Material	**Student Activity**

Lecture Notes

① Two forces 60N and 65N act on a screw at an angle of 25° and 85° from the base. Determine the magnitude and direction of their resultant.



Magnitude of Resultant force

$\Sigma H = 60 \cos 25° + 65 \cos 85°.$

$= 60 N.$

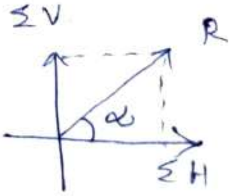
$\Sigma V = 60 \sin 25 + 65 \sin 85$

$= 90 N.$

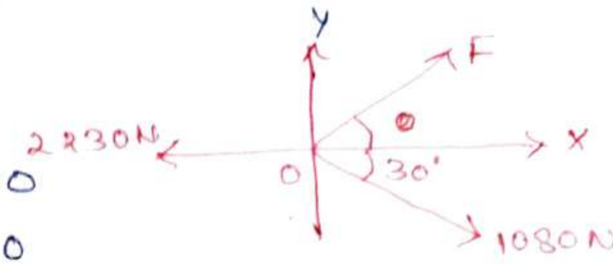
$\therefore R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{60^2 + 90^2} = 108.17 N.$

Direction: $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$= \tan^{-1}\left(\frac{90}{60}\right) = 56.31°.$



② The force system shown in figure has a resultant of 900N pointing up along the y-axis. Find the value of F and θ required to give this resultant.



Given $R = 900N (\uparrow)$

$\Sigma H = 0$

$F\cos\theta - 2230 + 1080\cos 30^0 = 0$

$F\cos\theta - 2230 + 935.31 = 0$

$F\cos\theta = 1294.69$ ----- ①

$\Sigma V = F\sin\theta - 1080\sin 30^0$

$900 = F\sin\theta - 540$

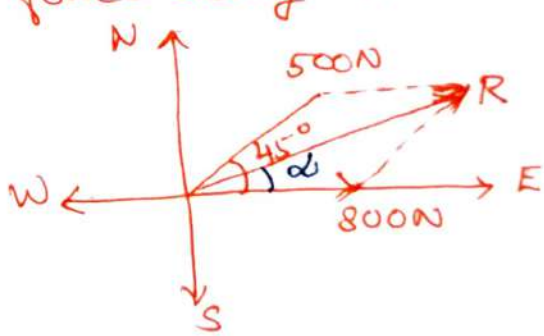
$F\sin\theta = 1440$ ----- ②

Solving ① & ②

$\frac{F\sin\theta}{F\cos\theta} = \frac{1440}{1294.69} \Rightarrow \tan\theta = 1.112 \Rightarrow \boxed{\theta = 48.04^0}$

$\therefore F = \frac{1294.69}{\cos 48.04} = 1936.38N$

③ Find the resultant of an 800N force acting towards eastern direction and a 500N force acting towards north eastern directions.



Let R be the resultant. Using parallelogram law of forces,

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{800^2 + 500^2 + 2 \times 800 \times 500 \times \cos 45^0}$

$= 1206.51N.$

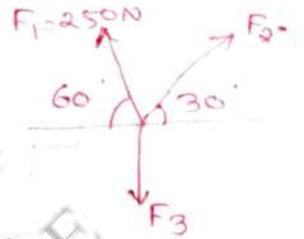
conditions of equilibrium: Equilibrium of particle in space

For equilibrium condition of force system, the resultant is zero. i.e. $R = 0$.

but, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\therefore \Sigma H = 0, \quad \Sigma V = 0$

(1) The forces shown are acting on a particle and keep the particle in equilibrium. The magnitude of force F_1 is 250 N. Find the magnitude of forces F_2 and F_3.



Method I: (By Lami's theorem)

Here there are only 3 concurrent forces acting outwards from a point. Hence Lami's theorem can be applied.

$\angle$ le opp. to F_1 is $\theta_1 = 90° + 30° = 120°$

$\theta_2 = 90 + 60° = 150°$

$\theta_3 = 180 - 60 - 30 = 90°$.

By Lami's theorem,

$$\frac{F_1}{\sin 120} = \frac{F_2}{\sin 150°} = \frac{F_3}{\sin 90°} \Rightarrow \frac{250}{\sin 120} = \frac{F_2}{\sin 150°} = \frac{F_3}{\sin 90°}$$

Solving we get, $F_2 = 144.33$ N. ; $F_3 = 288.67$ N.

Method II: [By equations of equilibrium]

The given system of forces is coplanar concurrent. Hence apply $\Sigma H = 0$ and $\Sigma V = 0$.

Applying $\Sigma H = 0 \ (\rightarrow +)$.

$F_2 \cos 30° - F_1 \cos 60° = 0$.

$$F_2 = \frac{250 \cos 60°}{\cos 30°}$$

$$\boxed{F_2 = 144.33 \text{ N}}$$

Applying $\Sigma V = 0 \ (\uparrow +)$

$F_1 \sin 60° + F_2 \sin 30 - F_3 = 0$

$250 \sin 60 + 144.33 \sin 30° = F_3$.

$$\therefore \boxed{F_3 = 288.67}$$

(2) Two wires are attached to a bolt in a foundation as shown. Determine the pull exerted by the bolt on the foundation (Anna 2003) April/May

$\Sigma H = 3600 \cos 25° - 6650 \cos 15°$

$= -3160$ N.

$\Sigma V = 3600 \sin 25° + 6650 \sin 15°$

$= 3242$ N.

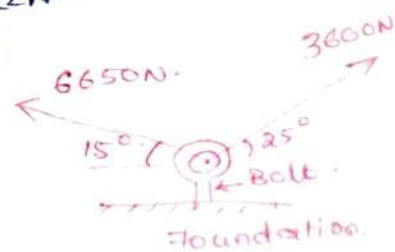
Resultant force $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(-3160)^2 + (3242)^2} = 4527$ N.

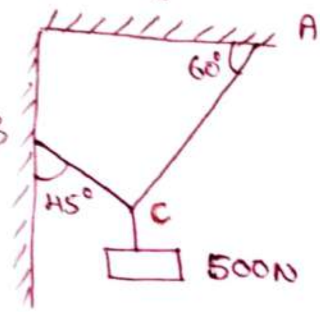
Direction $\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$

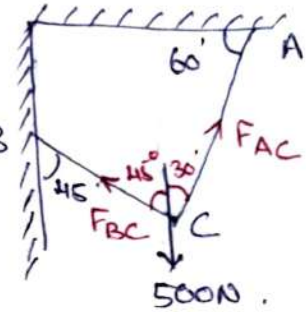
$\alpha = \tan^{-1}\left[\frac{3242}{3160}\right]$

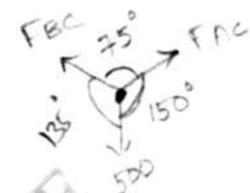
$$\boxed{\alpha = 45.73°}$$



(3) A weight of 500N is hung at a point C by a thin steel wire with supports at A and B. The system is in equilibrium, find the forces on the steel wire along AC and BC using Lami's theorem or otherwise







a) By Lami's theorem:

Let tension in the wire be F_{CA} and F_{CB}.

applying Lami's theorem:

$$\frac{F_{CA}}{\sin(180' - 45°)} = \frac{F_{BC}}{\sin(180° - 30')} = \frac{500}{\sin 75°}.$$

$$F_{CA} = \frac{500 \sin 45°}{\sin 75°} = \frac{353.6}{0.966} = 366\,N.$$

$$F_{BC} = \frac{500 \sin 30'}{\sin 75°} = \frac{250.0}{0.966} = 258.8\,N.$$

b) By method of resolution:

Resolving forces horizontally at C

$$F_{BC} \sin 45° = F_{AC} \sin 30°.$$

$$\frac{F_{BC}}{\sqrt{2}} = \frac{F_{AC}}{2}$$

$$\boxed{F_{AC} = \sqrt{2}\,F_{BC}}$$

Resolving forces vertically at C,

$$F_{BC} \cos 45° + F_{AC} \cos 30' = 500.$$

Sub. for F_{AC}, $$\frac{F_{BC}}{\sqrt{2}} + \sqrt{2}\,F_{BC}\frac{\sqrt{3}}{2} = 500$$

$$\frac{F_{BC}}{\sqrt{2}}\left[1 + \sqrt{3}\right] = 500.$$

$$\boxed{F_{BC} = 258.8\,N} \quad \boxed{F_{AC} = 360\,N}$$

UNIT II

Equilibrium of Rigid bodies

2.1 Introduction:- When some external forces (which may be concurrent or parallel are acting on a stationary body, the body may start moving or may start rotating about any point. But if the body does not start moving and also does not start rotating about any pt., then the body is said to be in equilibrium.

Equations of Equilibrium:- A stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces and " " " " moments of all the external forces about any point is zero.

$$\Sigma F = 0$$

$$\Sigma M = 0$$

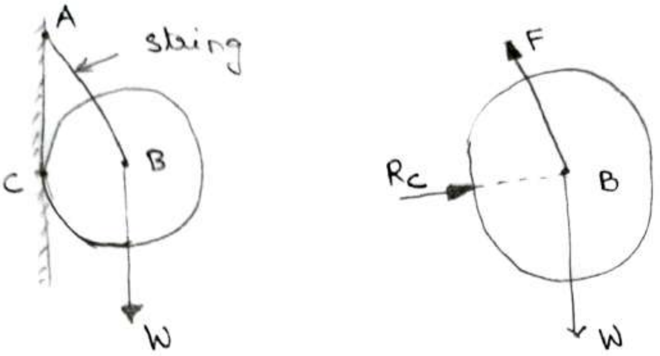
The forces are generally resolved into horizontal and vertical components.

$\therefore \Sigma F_x = 0 \quad ; \quad \Sigma F_y = 0.$

2.2 Free Body Diagram:- If we remove the support and replace it by the reaction R_A we get the free body diagram.

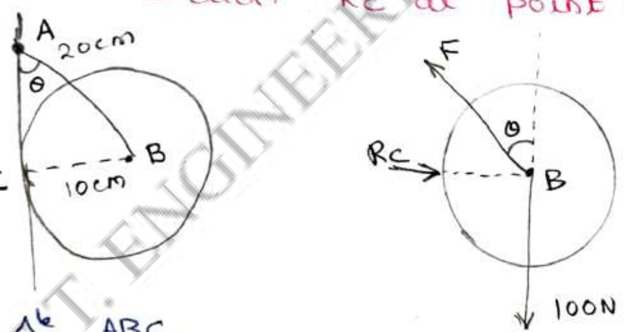
The pt. of application of the reaction R_A will be the pt. of contact A. To draw a FBD, the ball is completely isolated from its support and all forces acting on it is shown by vectors. (remove all the supports (like wall, floor, hinges or any other bodys and replace them by the reactions which these support exert on the body. Also the body should be completely isolated.

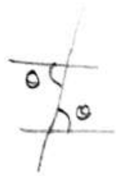
1. Draw the free body diagram of ball of weight W supported by a string AB and resting against a smooth vertical wall at C as shown in figure.



Here the magnitude of F and R_C is unknown.

2. A circular roller of weight 100N and radius 10cm hangs by a tie rod AB = 20cm and rests against a smooth vertical wall at C. Determine
   (i) the force F in the tie rod, and
   (ii) the reaction R_C at point C.





From Δ^{le} ABC,

$\sin\theta = \frac{BC}{AB} = \frac{10}{20} = 0.5$.

$\theta = \sin^{-1} 0.5 = 30^\circ$.

The free body diagram of roller is shown in figure in which,
R_C = Reaction at C.
F = Force in tie rod AB.

Free body diagram shows the equilibrium of the roller. Hence the resultant force in x-direction and y-direction should be zero.

$\Sigma f_x = 0$, we get $R_C - F\sin\theta = 0$.

$R_C = F\sin\theta$ . . . . (1)

For, $\Sigma F_y = 0$, we get

$$100 - F\cos\theta = 0.$$

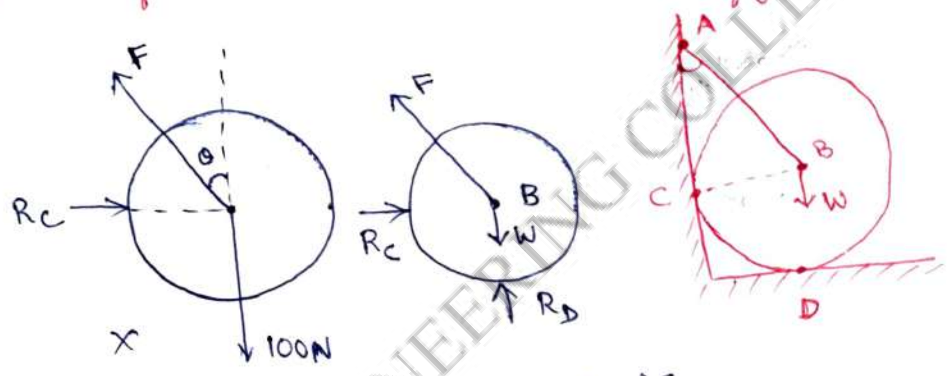
$$100 = F\cos\theta.$$

$$F = \frac{100}{\cos\theta} = \frac{100}{\cos 30^\circ} = 115.47\,N.$$

substituting the value of F in equation (1), we get

$$R_C = 115.47 \times \sin 30^\circ = 57.7\,N.$$

3. Draw the free body diagram of a ball of weight W, supported by a string AB and resting against a smooth vertical wall at C and also resting against a smooth horizontal floor at D as shown in figure.



(i) Reaction R_C at point C, normal to AC.

(ii) Force F in the direction of string.

(iii) wt. W of the wall,

(iv) Reaction R_D at pt. D, normal to horizontal surface.

The reactions R_C and R_D will pass thro' the centre of the ball i.e. thro' pt. B.

4. Two identical rollers, each of wt. 50N are supported by an inclined plane and a vertical wall as shown. Find the reaction at the pts. of supports A, B and C. Assume all the surfaces to be smooth.

Here R_A, R_B, R_C are reactions @ pts A, B and C.

F_{DE} and F_{ED} are the reactions between the two rollers and for equilibrium condition, $F_{DE} = F_{ED}$.

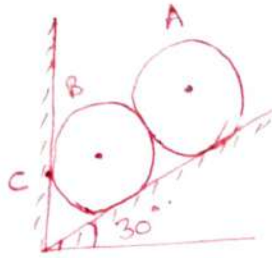
F_{ED} = Reaction from E to D, and F_{DE} .....

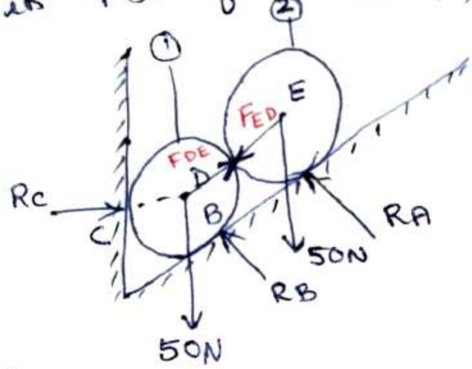
F_{ED} and F_{DE} are collinear and parallel to inclined plane.

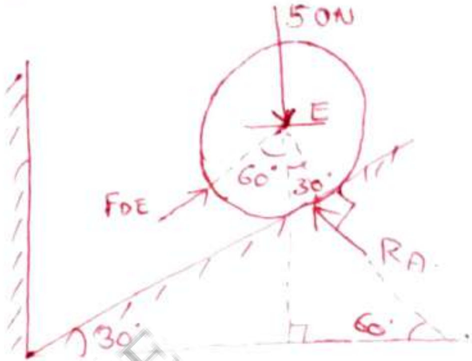
R_B & R_A is normal to inclined plane, passing thro' the ...

R_C is normal to wall, passing thro' centre of roller.

In FBD of roller (1), F_{DE} need not be considered. 11^{ly} in FBD of roller (2), F_{ED} need not be considered.



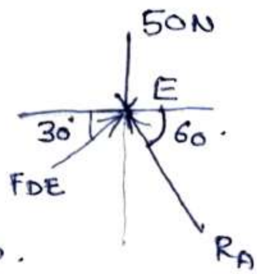




FBD of roller (2): F_{DE} is parallel to the inclined surface. FBD of roller (2).

$\Sigma H = 0 \Rightarrow F_{DE}\cos 30° - R_A\cos 60° = 0$ . . . ①

$\Sigma V = 0 \Rightarrow F_{DE}\sin 30° + R_A\sin 60 - 50 = 0$ . . . ②



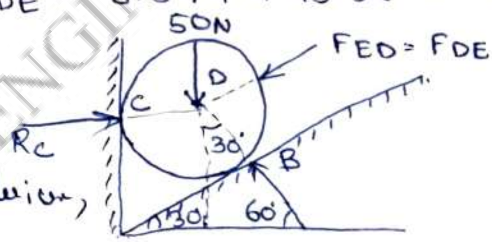
Solving ① & ②, we get $\boxed{F_{DE} = 0.577 R_A}$

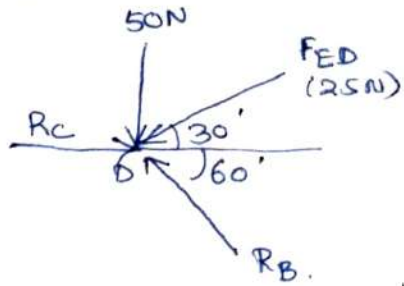
sub. in ②, $(0.577R_A \times 0.5) + (0.866 R_A) = 50$.

$1.154 R_A = 50 \Rightarrow R_A = 43.32\,N$.

$\therefore F_{DE} = 0.577 \times 43.32 = 25\,N$.

FBD of roller (1):





applying eqns. of equilibrium,

$\Sigma H = 0 \Rightarrow R_C - 25\cos 30° - R_B\cos 60° = 0$ . . . ①

$\Sigma V = 0 \Rightarrow R_B\sin 60° - 25\sin 30° - 50 = 0$ . . . ②

From equ. ②, $R_B\sin 60° = 50 + 25\sin 30° = 62.5$.

$$R_B = \frac{62.5}{\sin 60°} = 72.17\,N.$$

sub. in ①,

$R_C - 25\cos 30° - 72.17\cos 60° = 0$

$\therefore R_C = 57.73\,N$.

Result:

Reaction at A = 43.32 N.

Reaction at B = 72.17 N.

Reaction at C = 57.73 N.

Lecture No. 05 **UNIT I - STATICS OF PARTICLES**

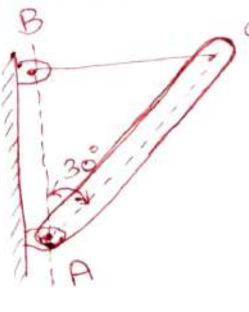
Topic(s) to be covered	

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

The bar AC, 10m long supports a load of 6000N as shown below, The cable BC is horizontal and 5m long. Determine the forces in the cable and the bar.



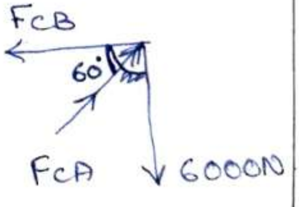
$\Sigma V = 0$.

$F_{CA} \sin 60° - 6000 = 0$

$F_{CA} = \frac{6000}{\sin 60°} = 6928.2 N$.

$\Sigma H = 0$.

$-F_{CB} + F_{CA} \cos 60° = 0$.



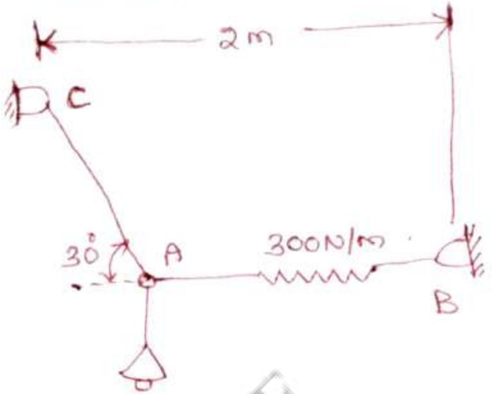
$F_{CB} = F_{CA} \cos 60°$
$= 6928.2 \cos 60° = 3464.1 N$.

$\therefore$ Force in the cable $= 3464.1 N$ (tensile).

coplanar concurrent

Force in the bar $= 6928.2 N$ (compressive).

Determine the length of cord AC in fig. below so that the 8 kg lamp is suspended in the position shown. The undeformed length of the spring AB is $l_{AB} = 0.4\,m$ and the spring has a stiffness of $k_{AB} = 300\,N/m$.



To find the force in the spring AB:

Applying $\Sigma V = 0$.

$F_{AC} \sin 30° - (8 \times 9.81) = 0$

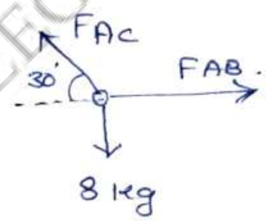
$\therefore \boxed{F_{AC} = 156.96\,N}$

Applying $\Sigma H = 0$.

$F_{AB} - F_{AC} \cos 30° = 0$

$F_{AB} = F_{AC} \cos 30 = 156.96 \cos 30°$

$\boxed{F_{AB} = 135.93\,N}$



In spring, we know, $F = KS$.

F = spring force, K = stiffness of the spring.

S = stretch of the spring.

$\therefore F_{AB} = K_{AB} S_{AB}$.

$\Rightarrow 135.93 = 300 \times S_{AB}$.

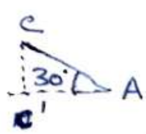
$\therefore \boxed{S_{AB} = 0.453\,m.}$

$\therefore$ Stretched length of AB, l'_{AB} = undeformed length + stretch of spring.

$l'_{AB} = l_{AB} + S_{AB}$

$= 0.4 + 0.453$

$= 0.853\,m.$



$\cos 30° = \frac{l_{C'A}}{l_{CA}}$

$l_{C'A} = l_{CA} \cos 30°$

$2m = l'_{AB}$ + Horz. length of AC.

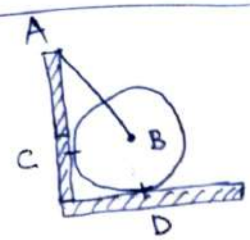
$2 = 0.853 + l_{AC} \cos 30°$.

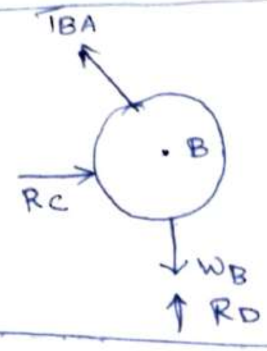
$\boxed{l_{AC} = 1.324\,m}$

Free Body Diagram.

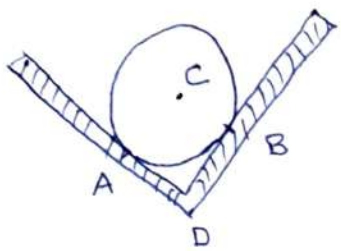
No.	Bodies under equilibrium	FBD

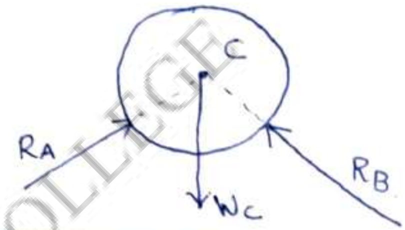
1.



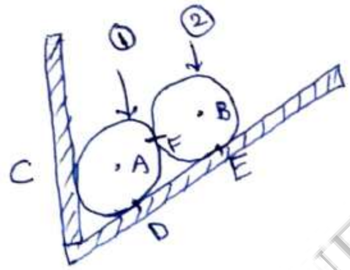


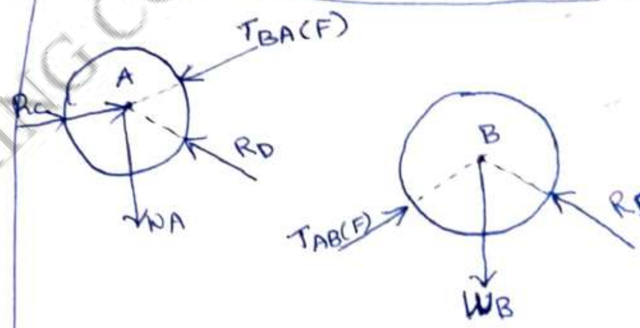
2.





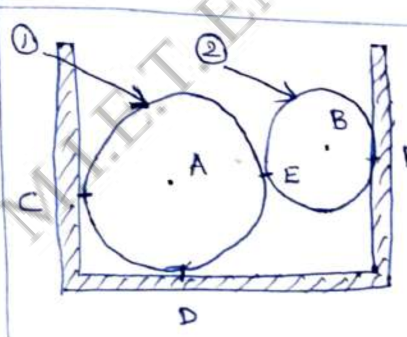
3.

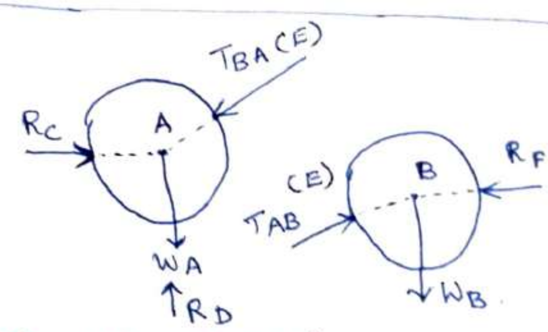




$T_{BA}(F) = T_{AB}(F)$

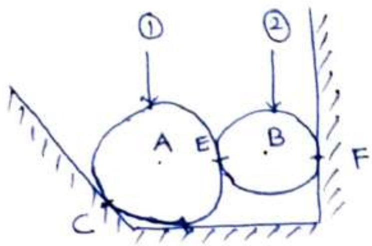
4.

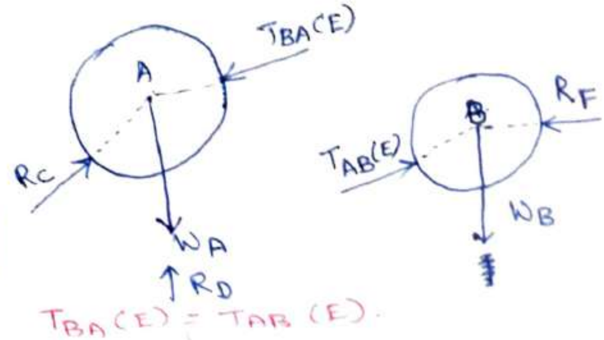




$T_{BA}(E) = T_{AB}(E)$

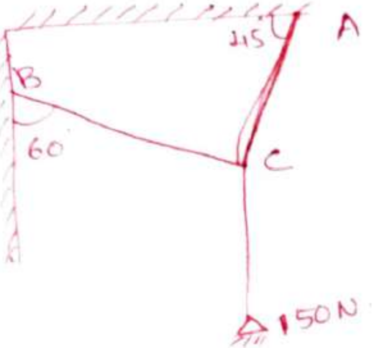
5.



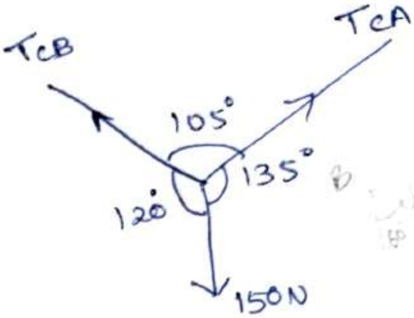


$T_{BA}(E) = T_{AB}(E)$.

An electric light fixture weighing 150N hangs from a pt C by two strings AC and BC in figure. Determine the fo[rces] in the string AC and BC.



⇒



The angle between T_{CB} and T_{CA} is
$= 360 - (120 + 135) = 105°$.

Method I (Lami's)

By applying Lami's equation at C.

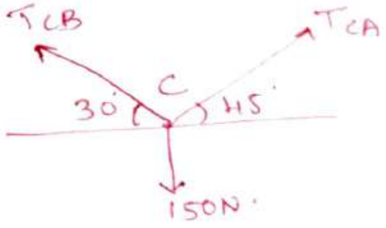
$$\frac{T_{CB}}{\sin 135°} = \frac{T_{CA}}{\sin 120°} = \frac{150}{\sin 105°}$$

$$\frac{T_{CA}}{\sin 120°} = \frac{650}{\sin 105°}$$

$$\therefore \; T_{CA} = \frac{150}{\sin 105°} \times \sin 120° = \frac{150}{0.97} \times 0.87 = 134.54\,N.$$

$$\frac{T_{CB}}{\sin 135°} = \frac{150}{\sin 105°} \Rightarrow T_{CB} = \frac{150 \sin 135°}{\sin 105°} = 109.81\,N.$$

Method II (equilibrium):-



$\Sigma H = 0$:

$T_{CA} \cos 45 - T_{CB} \cos 30 = 0.$

$0.707\, T_{CA} = 0.866\, T_{CB}$

$\boxed{T_{CA} = 1.225\, T_{CB}}$ .... (1).

$\Sigma V = 0$:

$T_{CA} \sin 45 + T_{CB} \sin 30 - 150 = 0.$

$T_{CA}\, 0.707 + 0.5\, T_{CB} - 150 = 0$

$1.225\, T_{CB} \times 0.707 + 0.50\, T_{CB} = 150.$

$1.366\, T_{CB} = 150.$

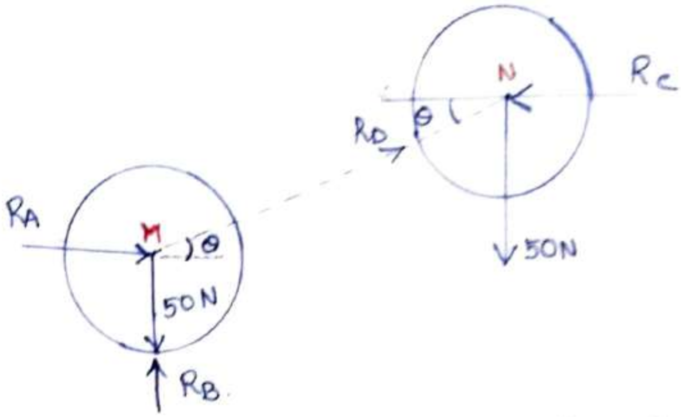
$\boxed{T_{CB} = 109.81N.}$ (2) substitute (2) in (1).

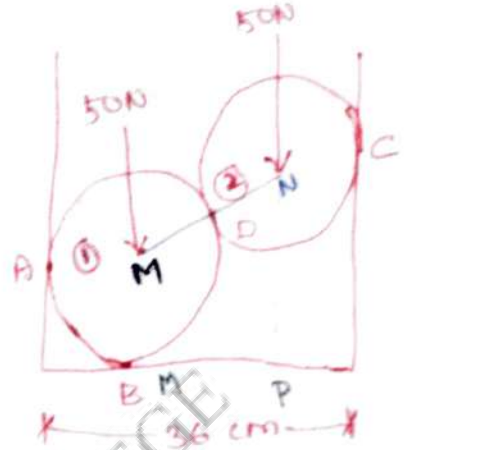
$T_{CA} = 1.224 \times 109.81$

$T_{CA} = 134.41N.$

Two rollers, each of wt 50N and of radius 10cm rest on a horz. channel of width 36cm as shown. Find the reaction on the point of contact A, B and C

Free Body diagram of rollers:





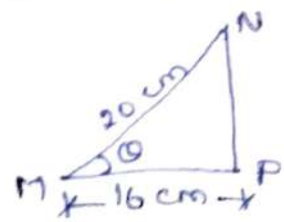
Let M be the centre of roller (1) and N be the centre of roller (2)

$MN =$ twice the radius of roller $= 2 \times 10cm = 20cm$.

$MP =$ Base − twice the radius.

$= 36 - (2 \times 10) = 16 cm$.

$\cos\theta = \frac{16}{20} \Rightarrow \theta = \cos^{-1}\left(\frac{16}{20}\right) = 36.87°$



considering free body diagram of Roller (2):

Applying $\Sigma H = 0$. ($\rightarrow$ +ve).

$R_D \cos\theta - R_C = 0 \Rightarrow R_D \cos 36.87 = R_C$ ······ (1).

Applying $\Sigma V = 0$ ($\uparrow$ +ve)

$R_D \sin\theta - 50 = 0 \Rightarrow R_D \sin 36.87 = 50$.

$\boxed{R_D = 83.33N}$

Sub. in (1), $\boxed{R_C = 66.66N}$

considering free body diagram of roller (1):

Applying $\Sigma H = 0$,

$R_A - R_D \cos\theta = 0 \Rightarrow R_A - R_D \cos 36.87 = 0$.

$R_A - 83.33 \cos 36.87 = 0$

$\boxed{R_A = 66.66N}$

Applying $\Sigma V = 0$, $\Rightarrow R_B - 50 - R_D \sin\theta = 0$.

$R_B - 50 - 83.33 \sin 36.87 = 0$.

$\therefore \boxed{R_B = 100N}$

$R_A = 66.68N (\rightarrow)$
$R_B = 100N (\uparrow)$
$R_C = 66.69N (\leftarrow)$

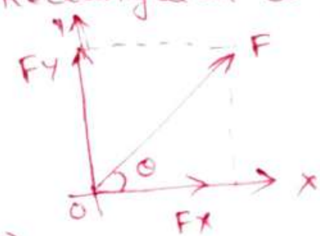
✓ Forces in space

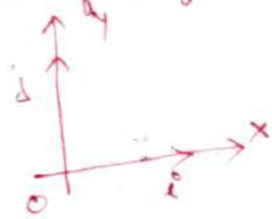
Resultant and equilibrium of particles in 3 dimensions

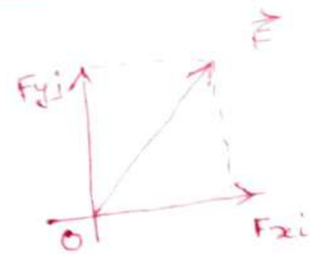
Vector approach: Finding scalar (trigonometrical method) components of a force in 3 dimensions is complicated. Hence vector approach is followed.

Vector approach for two-dimensional problem:

(1) Rectangular components of a force.







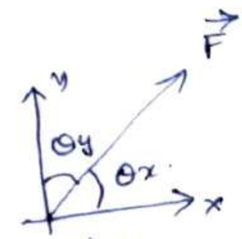
(a)

$$\vec{F} = \vec{F_x} + \vec{F_y}$$

$\vec{F_x} = F_x i \Rightarrow$ vector quantity along x-axis = scalar qty along x-axis × unit vector along x-axis.

$$\vec{F} = F_x i + F_y j$$

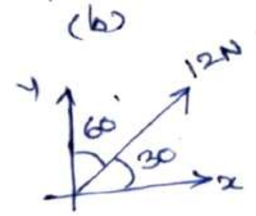
$$\boxed{\vec{F} = F\cos\theta i + F\sin\theta j}$$



(b)

Force vector interms of θ_x and θ_y.

$$\boxed{\vec{F} = F\cos\theta_x i + F\cos\theta_y j.}$$



Write the given force in vector form:

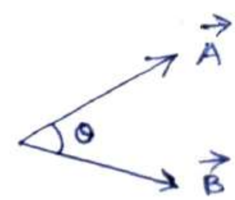
$$\vec{F} = F\cos\theta_x i + F\cos\theta_y j$$

$$= 12\cos 30\, i + 12\cos 60\, j.$$

$$\vec{F} = 10.392 i + 6j.$$

Important vector operations.

1. Vector addition and subtraction:



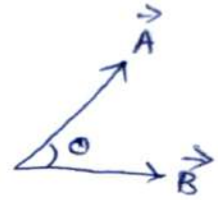
The sum of two or more vectors can be obtained by adding i and j components.

Let $\vec{A} = A_x i + A_y j$ and $\vec{B} = B_x i + B_y j$.

then $(\vec{A} + \vec{B}) = (A_x + B_x) i + (A_y + B_y) j$

and $(\vec{A} - \vec{B}) = (A_x - B_x) i + (A_y - B_y) j.$

(2) Dot Product: (or scalar product).



$$\vec{A}.\vec{B} = AB\cos\theta$$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y.$$

$\vec{A} = A_x i + A_y j$

$\vec{B} = B_x i + B_y j.$

Note: $i.i = 1.$ $\quad i.j = 0$

$j.j = 1$ $\quad j.i = 0.$

(3) cross product (or vector product).

Let $\vec{A} = A_x i + A_y j$ and $\vec{B} = B_x i + B_y j$

then $\vec{A}\times\vec{B} = |A|.|B|.\sin\theta$

$= AB\sin\theta.$

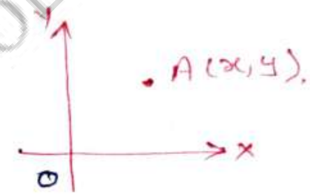
i) direction of $\vec{A}\times\vec{B}$ is ⟂r to the plane containing $\vec{A}$ & $\vec{B}$.

ii) magnitude $= AB\sin\theta.$

Note: $i \times i = 0$

$j \times j = 0.$

Position vector:

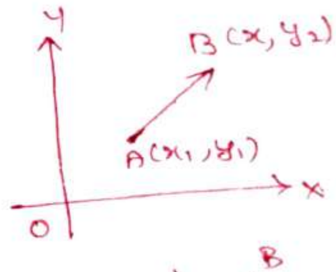


The position vector $\vec{r}$ of pt. A (x,y) w.r.t. O is written as $\vec{r} = xi + yj.$

and magnitude of position vector

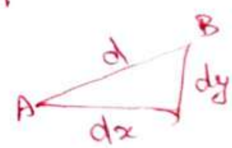
$$r = \sqrt{x^2+y^2}.$$

Force vector in terms of co-ordinates.



$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\theta_x = \cos^{-1}\left(\frac{x_2-x_1}{d}\right) \text{ and } \theta_y = \cos^{-1}\left(\frac{y_2-y_1}{d}\right)$$

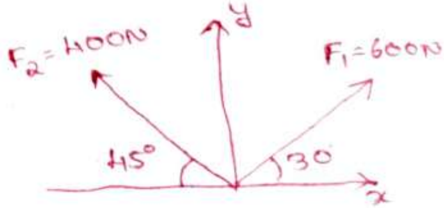


$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) \quad \left(\text{or } \frac{dx}{d}\right)$$

$$F_x = \frac{F\,dx}{d}.$$

① Find the resultant force of concurrent forces shown in fig.

Method I: Scalar approach.



Resolving forces horizontally.

$\Sigma H (\rightarrow +) = 600\cos 30° - 400\cos 45°$

$= 519.61 - 282.84 = 236.77N.$

Resolving forces vertically,

$\Sigma V (\uparrow +) = 600\sin 30° + 400\sin 45°$

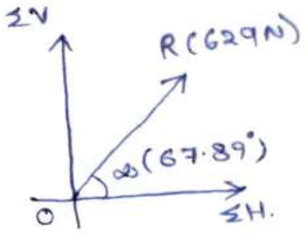
$= 300 + 282.84 = 582.84N.$

magnitude of resultant force

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(236.77)^2 + (582.84)^2}$$

$$\boxed{R = 629N}$$

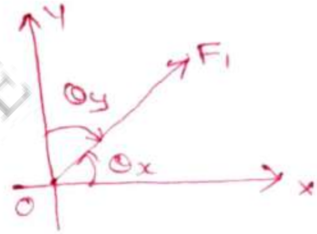


angle of resultant force with x-axis,

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{582.84}{236.77}\right) = 67.89°.$$

Method II : Vector approach

Write F_1 and F_2 in vector quantities.
(with i, j components).



Force $F_1 = 600N$.

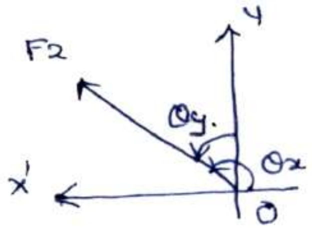
$$\vec{F_1} = F_1 \cos\theta_x i + F_1 \cos\theta_y j$$

$$= 600 \cos 30 i + 600 \cos 60 j.$$

$$\boxed{\vec{F_1} = 519.6 i + 300 j}$$

Force $F_2 = 400N$.

$\theta_x = \underset{(90+45)}{135°}$, $\theta_y = 45°$



$$\therefore \vec{F_2} = F_2 \cos\theta_x i + F_2 \cos\theta_y j.$$

$$= 400 \cos 135 + 400 \cos 45$$

$$\boxed{\vec{F_2} = -282.84 i + 282.84 j.}$$

Resultant force $\vec{R} = \vec{F_1} + \vec{F_2}$

$$\vec{R} = (519.6i + 300j) + (-282.84i + 282.84j)$$

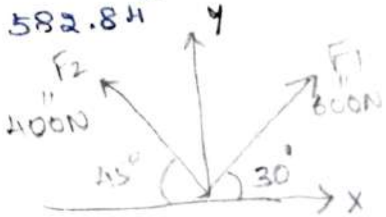
$$\vec{R} = (519.6 - 282.84) i + (300 + 282.84) j$$

$$\vec{R} = 236.76i + 582.84j.$$

Magnitude of resultant force

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{236.76^2 + 582.84^2}$$

$$\boxed{R = 629N}$$



Angle of resultant force with x-axis,

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$$

$$= \cos^{-1}\left(\frac{236.76}{629}\right) = 67.89°$$

Vector approach for a 3-Dimensional problem

$$\vec{F} = \vec{F_x} + \vec{F_y} + \vec{F_z}$$

$$\vec{F} = F_x i + F_y j + F_z k.$$

$F_x = F\cos\theta_x$; $F_y = F\cos\theta_y$ & $F_z = F\cos\theta_z$.

$$\vec{F} = (F\cos\theta_x) i + (F\cos\theta_y) j + (F\cos\theta_z) k.$$

$$|\vec{F}| = F = \sqrt{F_x^2 + F_y^2 + F_z^2}.$$

unit vector (λ):

$$\text{unit vector} = \frac{\text{vector Quantity}}{|\text{vector element}|}.$$

i unit vector along x-axis $= \dfrac{\text{vector quantity } \vec{F_x}}{\text{Magnitude } F_x}.$

$$\lambda = \frac{\text{Force vector } \vec{F}}{\text{Magnitude of } F} = \frac{\vec{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}.$$

$\vec{F}$ = Force x Unit vector along the force λ.

$$\boxed{\vec{F} = F\lambda}$$

$$\lambda = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \qquad \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & -3 & 4 \end{vmatrix} = 15i - 5j - 15k$$

$$|\vec{A} \times \vec{B}| = \sqrt{15^2 + 5^2 + 15^2}$$

$$= \frac{15i - 5j - 15k}{\sqrt{15^2 + 5^2 + 15^2}}$$

$$= 0.688i - 0.229j - 0.69k.$$

Position vector:

The position vector of point A w.r.t origin O is the vector $\vec{OA}$ represented as $\vec{r}$. The position vector $\vec{r}$ of point (x, y, z) in space is written as

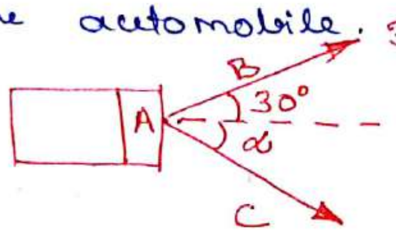
$$\vec{r} = xi + yj + zk.$$

and the magnitude of position vector

$$r = \sqrt{x^2 + y^2 + z^2}.$$

UNIT I - Home Work Problems

(1) A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope AB is 3750N, Determine by trigonometry the tension in rope AC and the value of α, so that the resultant force exerted at A is 6000N force directed along the axis of the automobile.

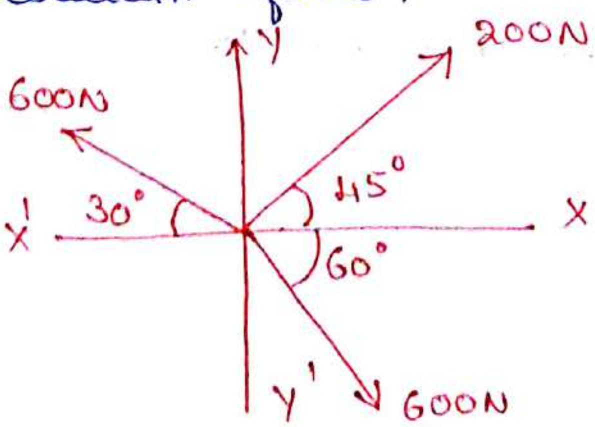


(2) A horizontal line PQRS is 12m long, where

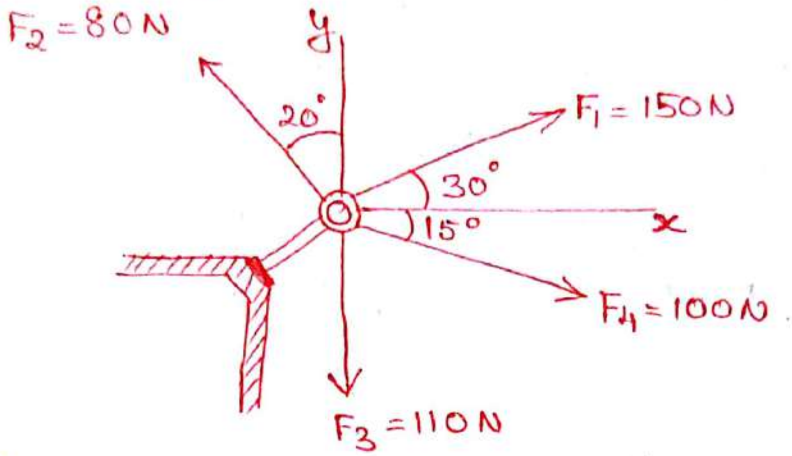
PQ = QR = RS = 4m.

Forces of 1000N, 1500N, 1000N and 500N act at P, Q, R, S respectively in downward direction. The line of action of these forces makes angle of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of resultant force.

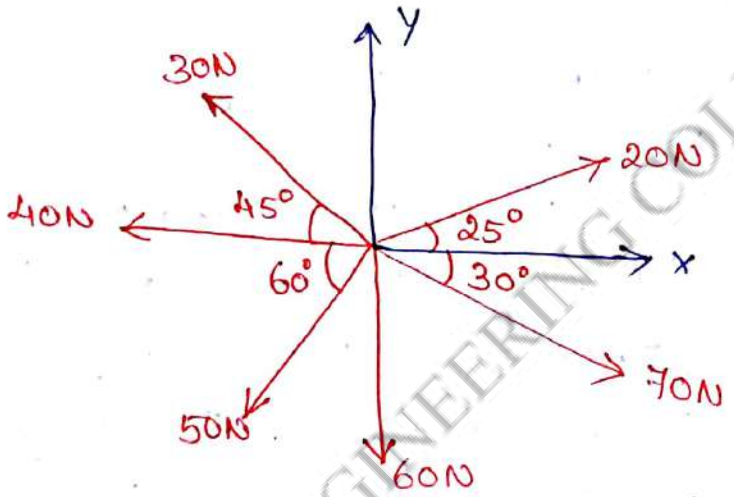
(3) Three coplanar forces are acting at a point as shown. Determine the magnitude and the direction of the resultant force.



(4) Four forces act on a bolt A as shown in figure. Determine the resultant of forces on the bolt. (7 mark)



(5) Find the resultant of the system of forces shown below. -(7 mark).



(6) The following forces act at a point:

i) 30N inclined at 32° towards North of East

ii) 15N towards North.

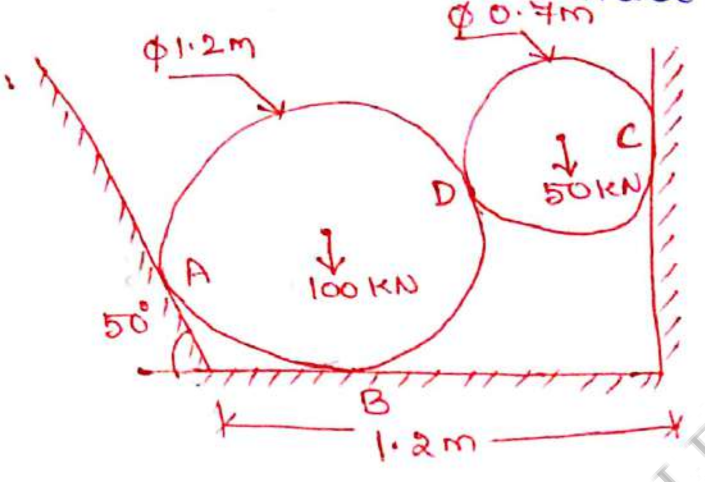
iii) 20N towards Northwest and

iv) 45N inclined at 45° towards South of west.

Find the magnitude and direction of the resultant force. -(7 mark)

7) a) State and prove Lami's theorem (7 mark)

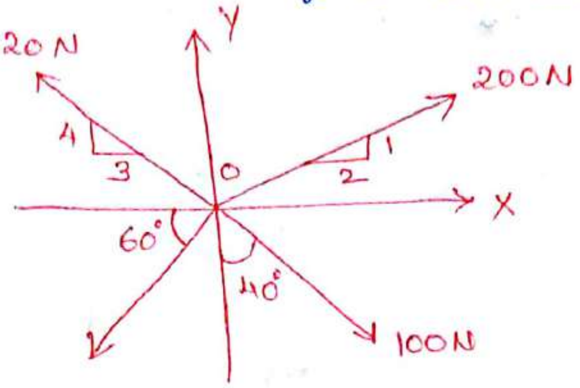
b) Two cylinders are kept in a channel as shown in fig. Determine the reaction at all the contact points A, B, C and D. Assume the contact surfaces are smooth



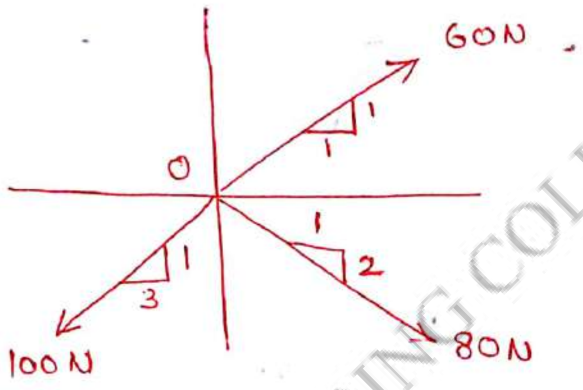
2 mark questions

1. Define Resultant force
2. What is meant by coplanar concurrent forces.
3. Write the principle of transmissibility.
4. State varignon's theorem.
5. State Lami's theorem.

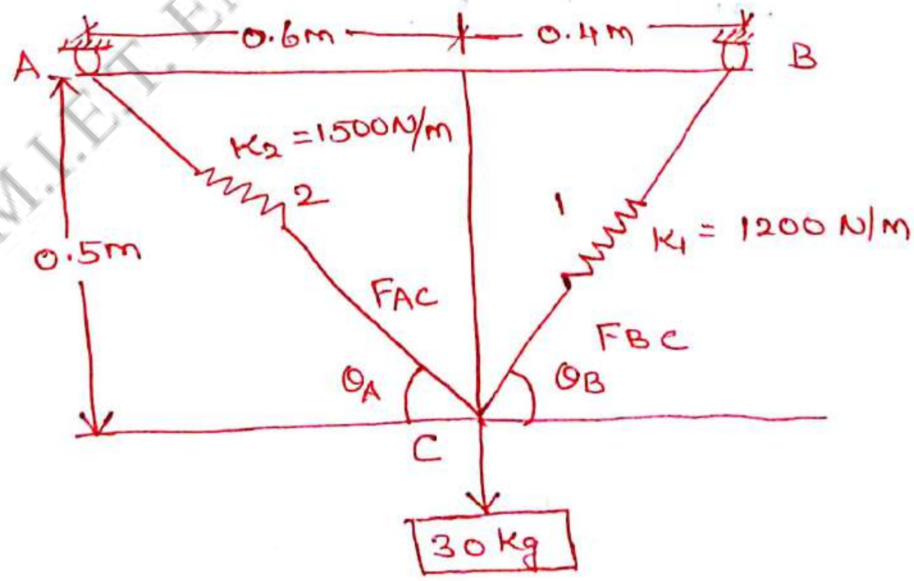
(8) A system of four forces acting on a body is shown in fig. below. Determine the resultant force and its direction.



(9) Determine the resultant of the force system shown.



(10) A 30 kg block is suspended by two springs having stiffness as shown. Determine the un-stretched length of each spring after the block is removed.



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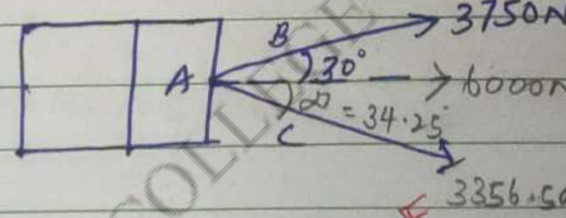
UNIT-1

① A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope AB is 3750N Determine by trigonometry the tension in rope AC and value of α, so that the resultant force exerted at A is 6000N force directed along the axis of the automobile.



$\Sigma H = 0$ ($\rightarrow$ +ve)

$$3750 \cos 30° + \overset{F\cos\alpha}{AC} = 6000$$

$$3750\,(0.866) + F\cos\alpha = 6000$$

$$3247.5 + F\cos\alpha = 6000$$

$$F\cos\alpha = 6000 - 3247.5$$

$$F\cos\alpha = 2752.5 \quad \text{—— ①}$$

$$R = \sqrt{H^2 + V^2} = 0$$

$\Sigma V = 0$ ($\uparrow$ +ve)

$$3750 \sin 30° - F\sin\alpha = 0$$

$$3750\,(0.5) - F\sin\alpha = 0$$

$$1875 - F\sin\alpha = 0$$

$$F\sin\alpha = 1875 \quad \text{—— ②}$$

$$\frac{\text{eqn } ②}{\text{eqn } ①} = \frac{\cancel{F}\sin\alpha}{\cancel{F}\cos\alpha} = \frac{1875}{2752.5}$$

$$\tan\alpha = 0.681$$

$$\alpha = \tan^{-1}(0.681)$$

$$\boxed{\alpha = 34.25°}$$

Sub in eqn ①

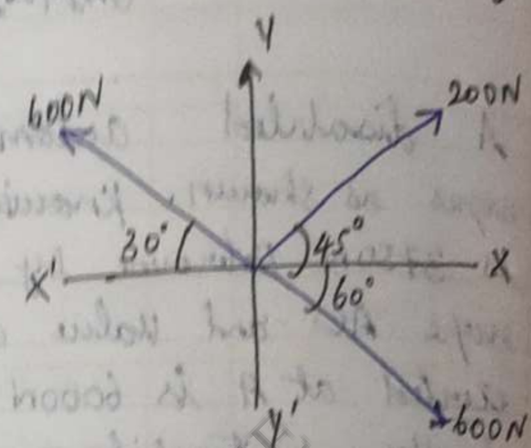
$$F\cos(34.25°) = 2752.41$$

$$F\,(0.82) = 2752.41$$

$$F = \frac{2752.41}{0.82}$$

$$\boxed{F = 3356.59\,N}$$

③ Three coplanar forces are acting at a point as shown. Determine the magnitude and the direction of the resultant force.



To find:

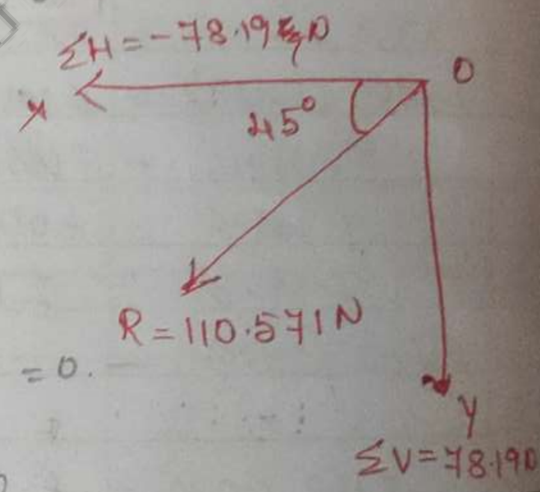
magnitulde, Resultant

Sol: $\underline{\underline{\Sigma H = 0}}$ ($\rightarrow$ +ve)

$-600 \cos 30° + 200 \cos 45° + 600 \cos 60° = 0$

$-519.61 + 141.42 + 300 = 0$

$\boxed{\Sigma H = -78.19 \text{ kN}}$ ✓



$\underline{\underline{\Sigma V = 0}}$ ($\uparrow$ +ve)

$600 \sin 30° + 200 \sin 45° - 600 \sin 60° = 0.$

$300 + 141.42 - 519.61 = 0$

$\boxed{\Sigma V = -78.19 \text{ kN}}$ ✓

$\underline{\underline{R}} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(-78.19)^2 + (-78.19)^2}$

$\boxed{R = 110.57 \text{ N}}$ ✓

Direction of Resultant:

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\alpha = \tan^{-1}\left(\frac{-78.19}{-78.19}\right)$

$\alpha = \tan^{-1}(1)$

$\boxed{\alpha = 45°}$ // ✓

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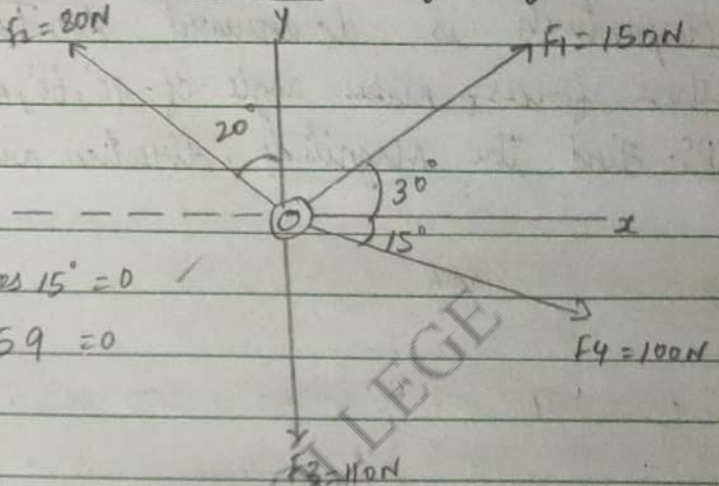
(4) Four Forces act on a bolt A as shown In fig. Determine the resultant of forces on the bolt.



$\Sigma H = 0$ ($\rightarrow$ +ve)

$$-80 \sin 20° + 150 \cos 30° + 100 \cos 15° = 0$$

$$-27.36 + 129.90 + 96.59 = 0$$

$$\boxed{\Sigma H = 199.13\ N}$$

$\Sigma V = 0$ ($\uparrow$ +ve)

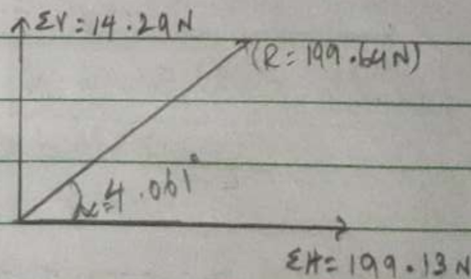
$$80 \cos 20° + 150 \sin 30° - 100 \sin 15° - 110 = 0$$

$$75.17 + 75 - 25.88 - 110 = 0$$

$$\boxed{\Sigma V = 14.29\ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(199.13)^2 + (14.29)^2}$$

$$\boxed{R = 199.64\ N}$$



$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$$

$$\alpha = \tan^{-1}\left[\frac{14.29}{199.13}\right] = \tan^{-1}[0.071]$$

$$\boxed{\alpha = 4.061°}$$

(6) The following forces act at a point:

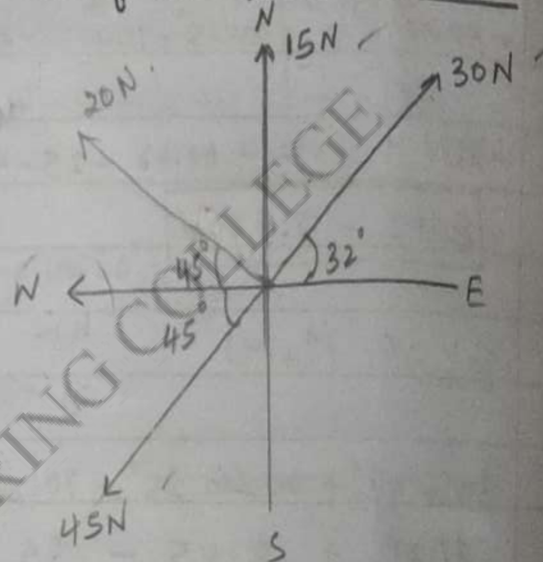
i) 30 N inclined at 32° towards North of East

ii) 15 N towards North

iii) 20 N towards ~~North~~ West

iv) 45 N inclined at 45° towards south of west.

Find the magnitude & direction of the resultant force.



$\Sigma H = 0$ ($\rightarrow$ +ve)

$30 \cos 32° - 45 \cos 45° - 20 \cos 45°$

$25.44 - 31.81 - 14.14$

$\boxed{\Sigma H = -20.51 \text{ N}}$

$\Sigma V = 0$ ($\uparrow$ +ve)

$+20 \sin 45° + 30 \sin 32° - 45 \sin 45° + 15 = 0$

$14.14 + 15.89 - 31.81 + 15 = 0$

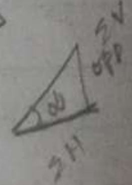
$\boxed{\Sigma V = 13.22 \text{ N}}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(-20.51)^2 + (13.22)^2}$

$\boxed{R = 24.40 \text{ N}}$

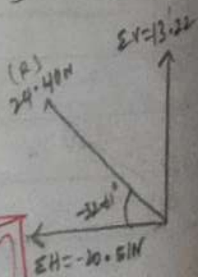
$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$ opp/adj

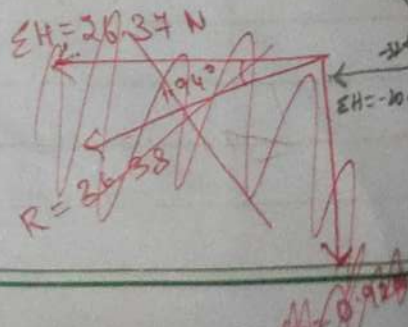


$\alpha = \tan^{-1}\left[\frac{13.22}{-20.51}\right]$

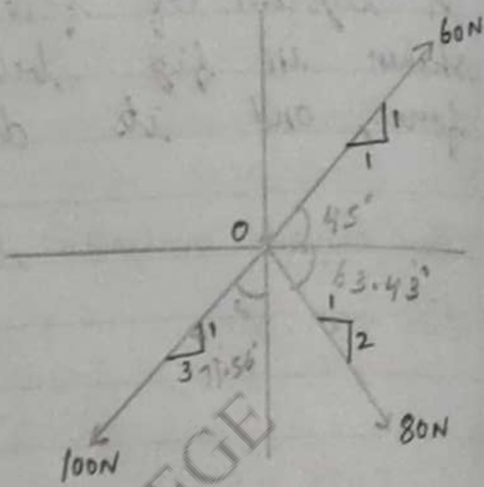
$\alpha = \tan^{-1}[-0.64]$

$\boxed{\alpha = -32.61°}$





⑨ Determine the resultant of the force shown system shown.



Sol: First we have to find "θ" for Force 60, 80 & 100N.

$$\overset{60N}{\alpha} = \tan^{-1}\left[\frac{1}{1}\right] = \tan^{-1}[1] = 45°$$

$$\overset{80N}{\alpha} = \tan^{-1}\left[\frac{2}{1}\right] = \tan^{-1}[2] = 63.43°$$

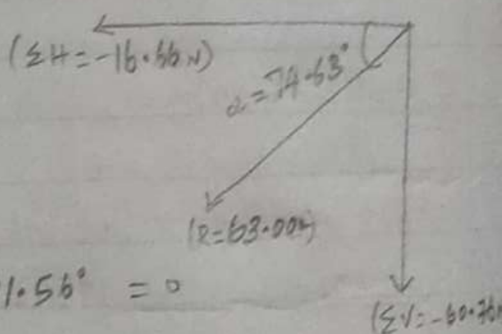
$$\overset{100N}{\alpha} = \tan^{-1}\left[\frac{3}{1}\right] = \tan^{-1}[3] = 71.56°$$

$\underline{\underline{\Sigma H = 0}}$ ($\rightarrow$ +ve)

$$-100 \sin 71.56° + 60 \cos 45° + 80 \cos 63.43° = 0$$

$$-94.86 + 42.42 + 35.78 = 0$$

$$\boxed{\Sigma H = -16.66\,N}$$



$\underline{\underline{\Sigma V = 0}}$ ($\uparrow$ +ve)

$$60 \sin 45° - 80 \sin 63.43° - 100 \cos 71.56° = 0$$

$$42.42 - 71.55 - 31.63 = 0$$

$$\boxed{\Sigma V = -60.76\,N}$$

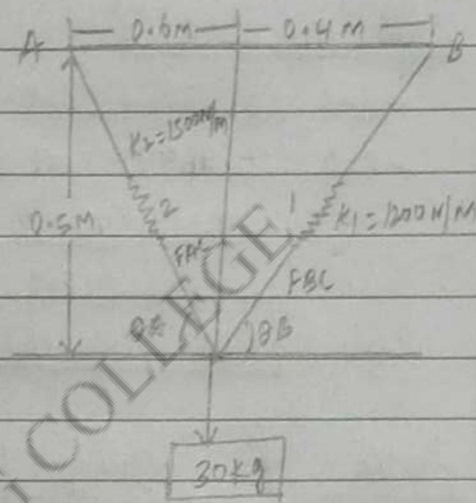
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-16.66)^2 + (-60.76)^2}$$

$$\boxed{R = 63.00\,N}$$

$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] = \tan^{-1}\left[\frac{-60.76}{-16.66}\right] = \tan^{-1}[3.64]$$

$$\boxed{\alpha = 74.63°}$$

(10) A 30 kg block is suspended by two springs having stiffness as shown. Determine the un-stretched length of each spring after the block is removed.



Find θA & θB

$\tan \theta A = \frac{opp}{adj} = \frac{0.5}{0.6} = 0.83$

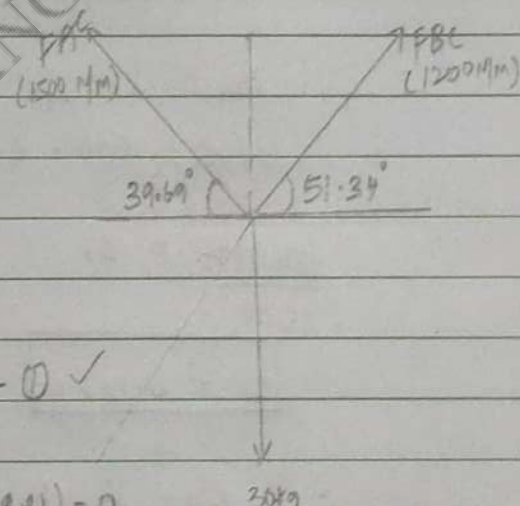
$\theta A = \tan^{-1}(0.83)$

$\theta A = 39.69°$

$\tan \theta B = \frac{opp}{adj} = \frac{0.5}{0.4} = 1.25$

$\theta B = \tan^{-1}(1.25)$

$\theta B = 51.34°$



$\Sigma H = 0$ ($\rightarrow$ +ve)

$-FAC \cos 39.69° + FBC \cos 51.34° = 0$

$-FAC(0.769) + FBC(0.624) = 0$ — (1)

$\Sigma V = 0$ ($\uparrow$ +ve)

$FAC \sin 39.69° + FBC \sin 51.34° - (30 \times 9.81) = 0$

$FAC(0.638) + FBC(0.780) = 294.3$ — (2)

$-FAC(0.769) + FBC(0.624) = 0 \rightarrow$ (1)

$FAC(0.638) + FBC(0.780) = 294.3 \rightarrow$ (2)

(1) × 0.638 $\quad -FAC(0.490) + FBC(0.398) = 0$

(2) × 0.769 $\quad FAC(0.490) + FBC(0.599) = 226.31$

$FBC(0.997) = 226.31$

$$F_{BC} = \frac{226.31}{0.997}$$

$$\boxed{F_{BC} = 226.99}$$

Sub in eqn (1)

$$-F_{AC}\,(0.769) + (226.99)\,(0.624) = 0$$

$$-F_{AC}\,(0.769) + 141.64 = 0$$

$$F_{AC} = \frac{141.64}{0.769} = 184.187$$

$$\boxed{F_{AC} = 184.187}$$

Force in Spring $(F) = k \cdot \Delta$

$$\therefore\ F_{AC} = k_1 \cdot \Delta_1$$

$$184.187 = 1500 \cdot \Delta_1$$

$$\frac{184.187}{1500} = \Delta_1$$

$$\boxed{\Delta_1 = 0.122\,m}$$

$$\therefore\ F_{BC} = k_2 \cdot \Delta_2$$

$$226.99 = 1200 \cdot \Delta_2$$

$$\frac{226.99}{1200} = \Delta_2$$

$$\boxed{\Delta_2 = 0.189\,m}$$

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Stretched length of spring be (1) & (2)

$$L = \sqrt{b^2+h^2}$$

$$L_1 = \sqrt{(0.6)^2+(0.5)^2} = 0.78\,M$$

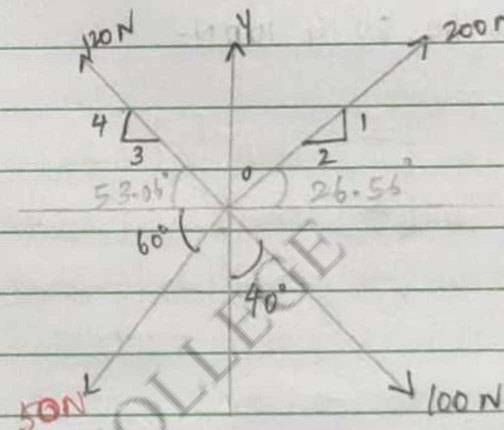
$$L_2 = \sqrt{(0.4)^2+(0.5)^2} = 0.64\,M$$

Unstretched length of spring are $(L_1-\Delta_1)$ & $(L_2-\Delta_2)$

$$\therefore (L_1-\Delta_1) \Rightarrow (0.78-0.122) = 0.658\,m$$

$$\therefore (L_2-\Delta_2) \Rightarrow (0.64-0.189) = 0.451\,m$$

(8) A system of four forces acting on a body is shown in fig below. Determine the resultant force and its direction.



Find θ for 120N, 200 N

120N = 2

$\alpha = \tan^{-1}\left[\frac{4}{3}\right]$

$= \tan^{-1}[1.33]$

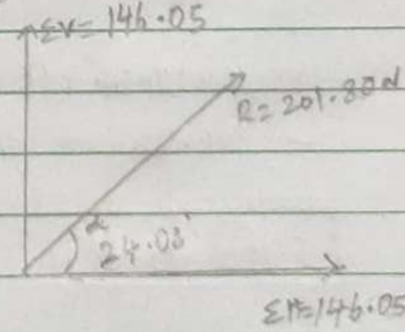
$\boxed{\alpha = 53.06^\circ}$

200N = 1

$\alpha = \tan^{-1}\left[\frac{1}{2}\right]$

$= \tan^{-1}[0.5]$

$\boxed{\alpha = 26.56}$



$\Sigma H = 0$ ($\rightarrow$ +ve)

$-120\cos 53.06^\circ + 200\cos 26.56^\circ + 100\sin 40^\circ - 50\cos 60^\circ$

$-72.11 + 178.89 + 64.27 - 25$

$\boxed{\Sigma H = 146.05\ N}$

$\Sigma V = 0$ ($\uparrow$ +ve)

$120\sin 53.06^\circ + 200\sin 26.56^\circ - 100\cos 40^\circ - 50\sin 60^\circ = 0$

$95.91 + 89.42 - 76.60 - 43.30$

$\boxed{\Sigma V = 65.43\ N}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(146.05)^2 + (65.43)^2}$

$\boxed{R = 160.036\ N}$

$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] = \tan^{-1}\left[\frac{65.43}{146.05}\right] = \tan^{-1}[0.447]$

$\boxed{\alpha = 24.08^\circ}$

(2) A horizontal line PQRS is 12m long where $PQ = QR = RS = 4m$.

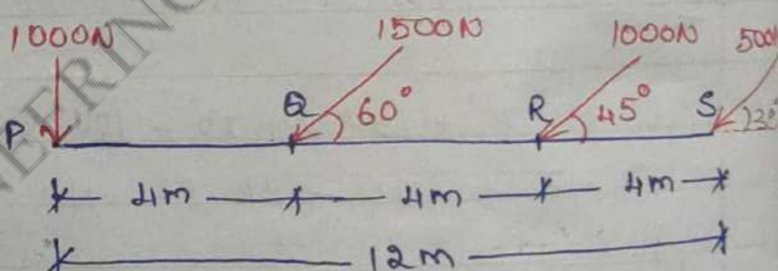
Force of 100 N, 1500 N, 1000N and 500N act at P, Q, R, S respectively is downward direction. The line of action of these forces makes angle of 90°, 60°, 45° and 30° respectively with PS. Find the Magnitude, direction and position of resultant force.

$\Sigma H = 0$ ($\rightarrow$ +ve)

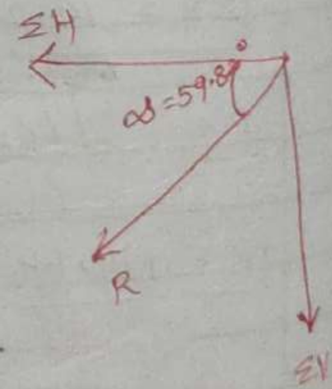
$$-1500\cos 60° - 1000\cos 45° - 500\cos 30° = 0$$

$$-750 - 707.10 - 433.01 = 0$$

$$\boxed{\Sigma H = -1890.11\,N}$$



$\Sigma V = 0$ ($\uparrow$ +ve)



$$-1500\sin 60° - 1000\sin 45° - 500\sin 30° - 1000 = 0$$

$$-1299.038 - 707.106 - 250 - 1000 = 0$$

$$\boxed{\Sigma V = -3256.14\,N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-1890)^2 + (-3256.14)^2}$$

$$\boxed{R = 3764.91\,N}$$

$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] = \tan^{-1}\left[\frac{-3256.14}{-1890.11}\right]$$

$$\alpha = \tan^{-1}[1.72]$$

$$\boxed{\alpha = 59.82°}$$

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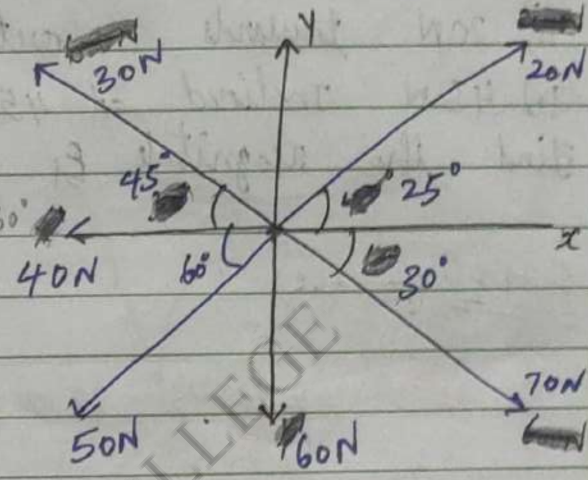
5) Four Forces act on a bolt A as shown in fig. Determine resultant of forces on the bolt.



$\underline{\Sigma H = 0}$ $(\rightarrow +ve)$

$-30\cos 45^\circ + 20\cos 25^\circ + 70\cos 30^\circ - 50\cos 60^\circ - 40$

$-21.21 + 18.12 + 60.62 - 25 - 40$

$\boxed{\Sigma H = -7.47\ N}$

$\underline{\Sigma V = 0}$ $(\uparrow +ve)$

$30\sin 45^\circ + 20\sin 25^\circ - 70\sin 30^\circ - 50\sin 60^\circ - 60 = 0$

$21.21 + 8.45 - 35 - 43.30 - 60 = 0$

$\boxed{\Sigma V = -108.64\ N}$

$\underline{R} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

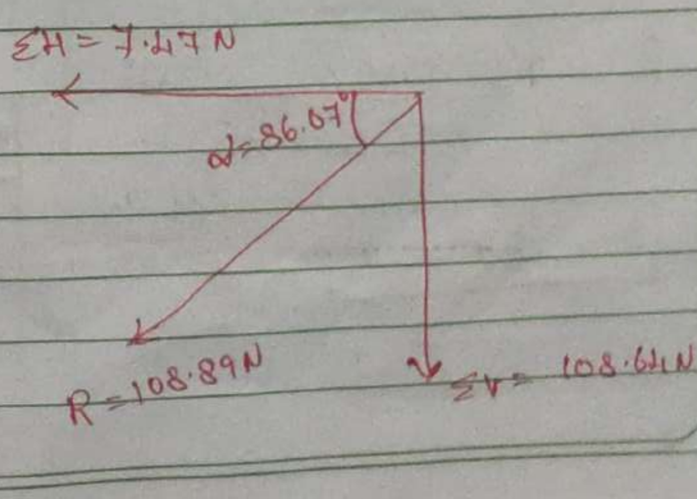
$= \sqrt{(-7.47)^2 + (-108.64)^2}$

$\boxed{R = 108.89\ N}$

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

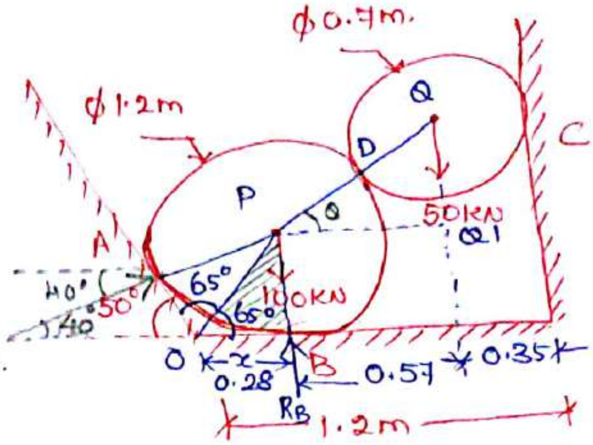
$= \tan^{-1}\left(\frac{108.64}{7.47}\right)$

$\boxed{\alpha = 86.07^\circ}$



7b). Consider Δ^{le} OPB;

$$\tan 65^\circ = \frac{PB}{OB} = \frac{opp.}{hyp} = \frac{0.6}{x}$$



$$x = \frac{0.6}{2.144} = 0.28$$

$$PQ' = 1.2 - 0.28 - 0.35 = 0.57m.$$

Consider Δ^{le} PQQ':

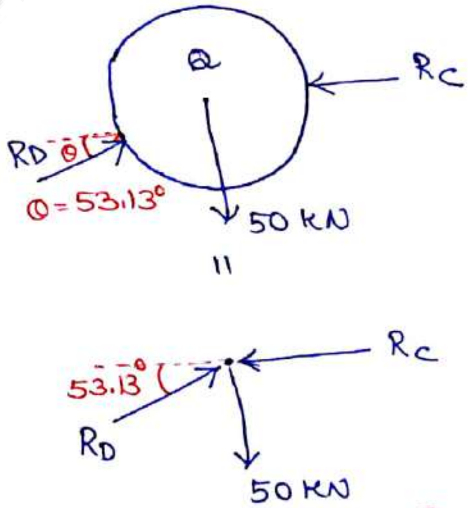
$$\cos\theta = \frac{adj}{hyp} = \frac{0.57}{0.95} = 0.6.$$

$$\theta = \cos^{-1}(0.6) \Rightarrow \boxed{\theta = 53.13^\circ}$$

Free body diagram of roller Q:

$\Sigma V = 0$ ($\uparrow$ +ve).

$$-50 + R_D \sin 53.13^\circ = 0$$



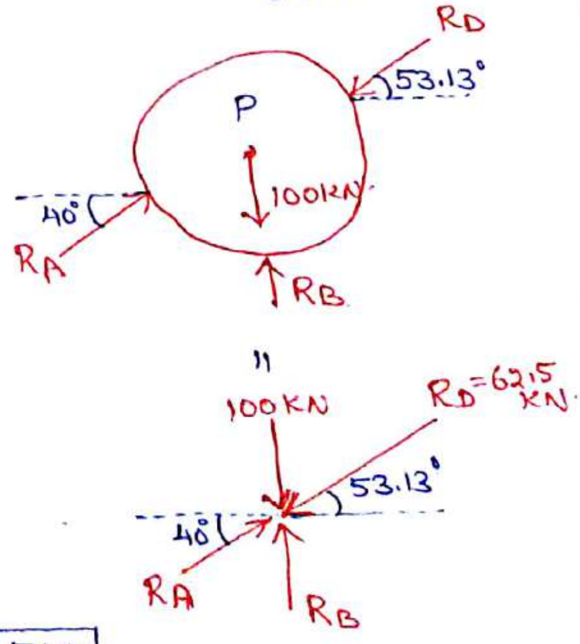
$$\boxed{R_D = 62.5 \text{ KN}}$$

$\Sigma H = 0.$ ($\rightarrow$ +ve).

$$R_D \cos 53.13^\circ - R_C = 0$$

$$\boxed{R_C = 37.5 \text{KN}}$$

Free Body diagram of roller P:



$\Sigma H = 0$ ($\rightarrow$ +ve).

$$-R_D \cos 53.13^\circ + R_A \cos 40^\circ = 0$$

$$-62.5 \cos 53.13^\circ + R_A \cos 40^\circ = 0.$$

$$R_A = \frac{37.5}{\cos 40^\circ} = 48.95 \simeq 49 \text{ KN}.$$

$\Sigma V = 0$ ($\uparrow$ +ve).

$$-R_D \sin 53.13^\circ - 100 + R_A \sin 40^\circ + R_B = 0.$$

$$-50 - 100 + 49 \sin 40^\circ + R_B = 0$$

$$-50 - 100 + 31.5 + R_B = 0.$$

$\boxed{R_B = 118.5 \text{ KN.}}$ $\boxed{R_A = 49 \text{KN.}}$ $\boxed{R_C = 37.5 \text{KN}}$

Lecture No. 10 **UNIT II – EQUILIBRIUM OF RIGID BODIES**

Topic(s) to be covered	Resultant of forces and its direction.

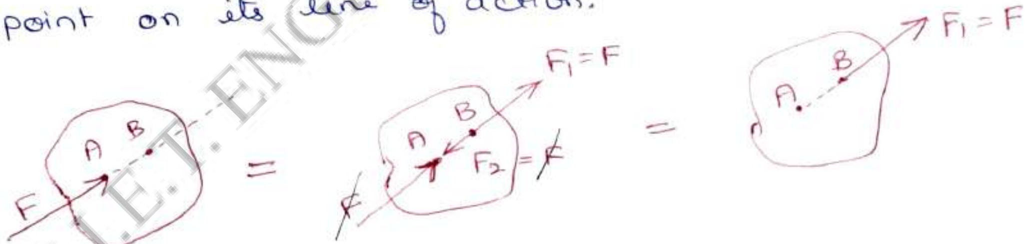
	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
Lo1	understand Principle of transmissibility & varignon's theorem.	L3
Lo2	find resultant and its direction.	

Teaching Learning Material	Student Activity
chalk and Talk	Learn and solve.

Lecture Notes

Principle of transmissibility of forces:

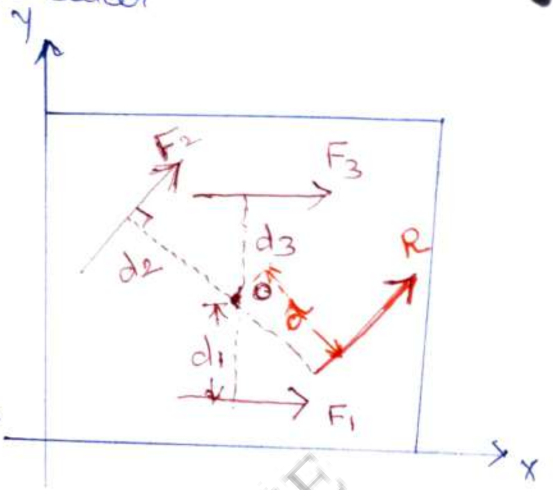
If a force acts at any point on a rigid body it may also be considered to act at any other point on its line of action.



Varignon's theorem:

The algebric sum of moments of any number of forces about any point in their plane is equal to the moment of their resultant about the same point. Varignon's theorem is also known as theorem of three moments.

consider a rigid body subjected to three coplanar forces F_1, F_2 and F_3 at perpendicular distances d_1, d_2 and d_3 from pt. O. Let the resultant force R is at a distance 'd' from 'O'.



From Varignon's theorem,

Sum of the moments of all the forces F_1, F_2 and F_3 about 'O' is equal to the moment of resultant for R about the same pt. 'O'

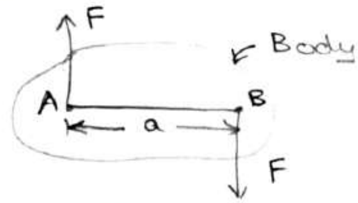
i.e. $F_1 d_1 + F_2 d_2 + F_3 d_3 = R \cdot d$

Moment of a couple:

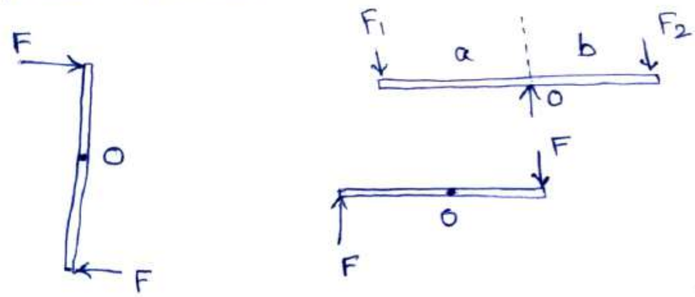
The perpendicular distance between the parallel forces is known as arm of the couple. The moment of the couple is the product of either one of the forces and perpendicular distance between the forces.

$\therefore$ Moment of the couple

$$M = F \times a.$$



※ Explain clockwise and anticlockwise moment for:



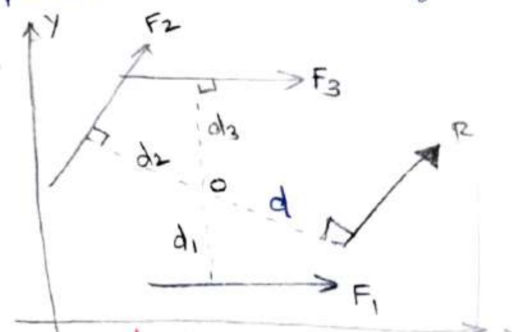
clockwise '+ve'

anticlockwise '−'ve.

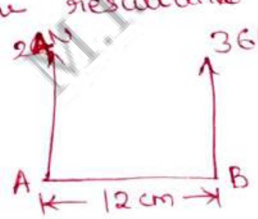
Varignon's theorem:

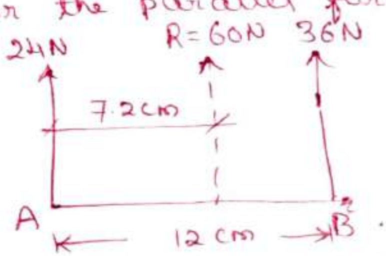
The algebric sum of the moments of any number of forces about any pt. in their plane = moment of their resultant about the same point.

$$F_1 d_1 + F_2 d_2 + F_3 d_3 = R.d$$



1. Find the resultant force for the parallel force system.





magnitude of resultant force = $R = 24+36 = 60N$.

Direction of resultant force upwards ↑ ∵ R is +ve.

Location of resultant force :-

$$\Sigma M_A = (24 \times 0) - (36 \times 12) = -432 \text{ N cm.}$$

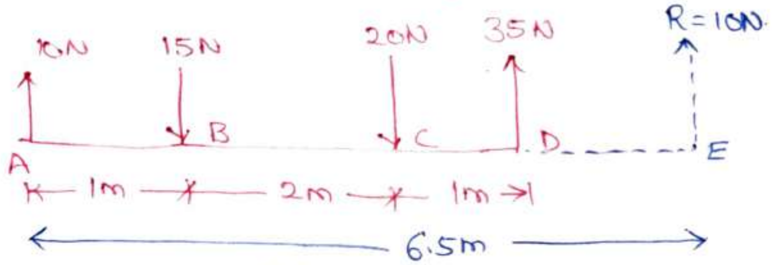
(− for anticlockwise)

moment of resultant about A,

$$R \times x = 432 \text{ N cm.}$$

$$x = \frac{432}{60} = 7.2 \text{ cm.} \quad \therefore \boxed{x = 7.2 \text{ cm}}$$

2. Four parallel forces of magnitude 10N, 15N, 20N and 35N are shown in figure. Determine the magnitude and direction of the Resultant. Find the distance of the resultant from pt A.



magnitude of Resultant Force @E, $R = 10-15-20+35 = 10N \uparrow$

Direction :- upwards.

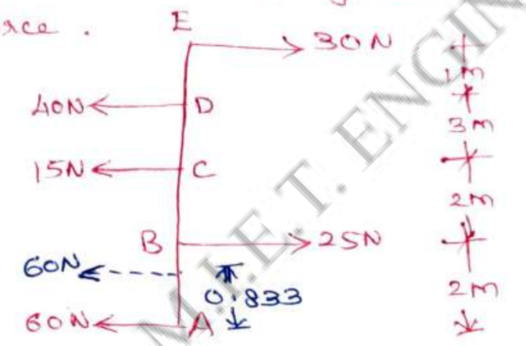
Location of Resultant Force :-

$$-Rx = (10\times0)+(15\times1)+(20\times3)-(35\times4)$$

$$-10\times x = -65\,Nm$$

$$\boxed{x = 6.5\,m}$$

3. Determine the magnitude direction and location of Resultant force.



left '−'ve
right '+'ve.

magnitude of resultant force

$= 30-40-15+25-60$

$= -60N$ (left).

let us find location of resultant force w.r.t. pt. A.

$$R.x = (30\times8)-(40\times7)-(15\times4)+(25\times2)+(60\times0)$$

$$-60x = -50\,Nm.$$

$$\boxed{x = 0.833\,m}$$

Resultant force of non-concurrent & non parallel forces.

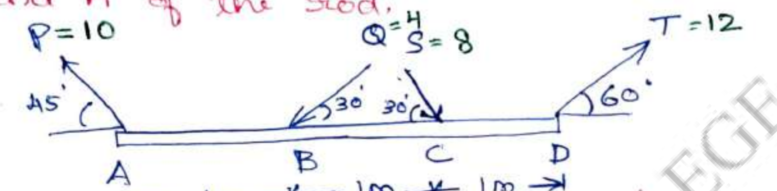
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

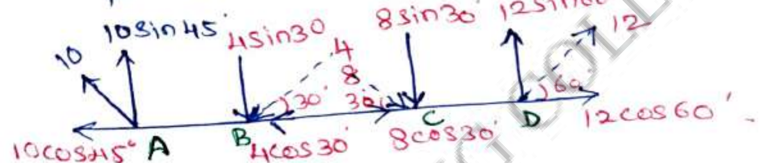
Direction of resultant force, $\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$.

Location of resultant force,

$$\Sigma M = R \times x.$$

4. ABCD is a weightless rod under the action of four forces, P, Q, S and T. If P = 10N, Q = 4N, S = 8N and T = 12N, calculate the resultant and mark the same in direction w.r.t. the end A of the rod.





$$(\rightarrow +)\quad \Sigma H = -10\cos 45° - 4\cos 30° + 8\cos 30° + 12\cos 60°.$$

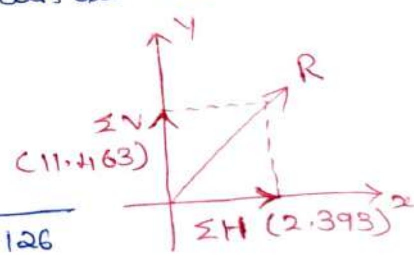
$$= -7.071 - 3.464 + 6.928 + 6 = 2.393\ N$$

Algebric sum of vertical components,

$$(\uparrow +)\quad \Sigma V = 10\sin 45° - 4\sin 30° - 8\sin 30° + 12\sin 60°$$

$$= 7.071 - 2 - 4 + 10.392 = 11.463\ N.$$

ΣH and ΣV are drawn on co-ordinate axes.



magnitude of resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

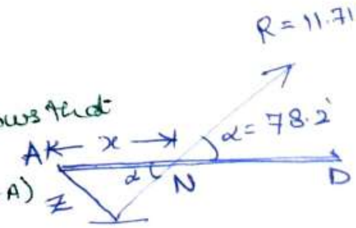
$$= \sqrt{(2.393)^2 + (11.463)^2} = \sqrt{137.126}$$

$$= 11.71\ N.$$

Direction of Resultant force,

$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] = \tan^{-1}\left[\frac{11.463}{2.393}\right] = 78.2°.$$

Location of resultant force :-



$$R \times Z = (4\sin 30° \times 1) + (8\sin 30 \times 2) - (12\sin 60° \times 3)$$

$$+11.71 \times Z = 2 + 8 - 31.176 = -21.176\ Nm.$$ (-ve sign shows that net moment is anticlockwise about A)

$$\therefore Z = \frac{-21.176}{+11.71} = 1.808\ m.$$

Hence R should be taken on the right of A). find x from rt. angled $\triangle$le

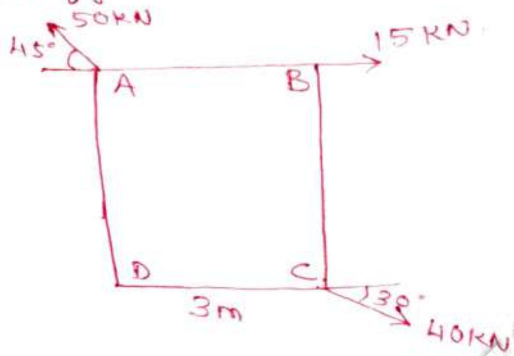
Alternate method.

ΣM_A = Moment of ΣV alone

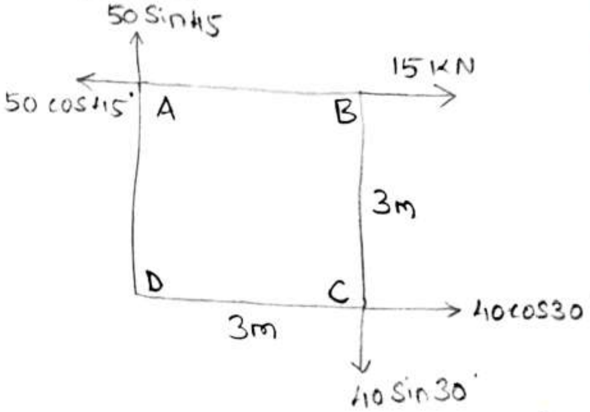
$\Sigma M_A = (\Sigma V \times x)$

$$\therefore x = \frac{\Sigma M_A}{\Sigma V} = \frac{21.176}{11.463} = 1.847\,m.$$

5. Determine the magnitude and line of action of the resultant of forces shown in figure.



Solution



Step 1: Resolve all inclined forces into horizontal and vertical components.

Now, Algebric sum of horizontal forces,

$\Sigma H = 15 + 40\cos 30 - 50\cos 45°$

$= 15 + 34.64 - 35.35 = 14.286$ KN.

Algebric sum of vertical forces,

$\Sigma V = 50\sin 45° - 40\sin 30°$

$= 35.35 - 20 = 15.35$ KN.

magnitude of Resultant force, $R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= \sqrt{(14.286)^2 + (15.35)^2}$

$= 20.97$ KN.

Direction of resultant force,

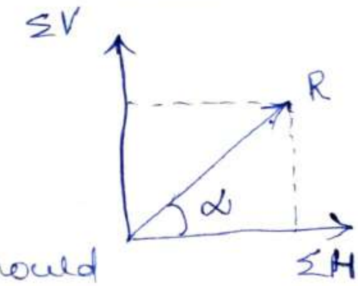
$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{15.35}{14.286}\right) = 47.06°$$

<u>Location of Resultant force:</u>

Let us locate resultant force from pt. A. Let x is the ⊥r distance from pt. A.

$\Sigma M_A = (40 \sin 30° \times 3) - (40 \cos 30° \times 3)$

$= 60 - 103.92 = -43.92$ KNm

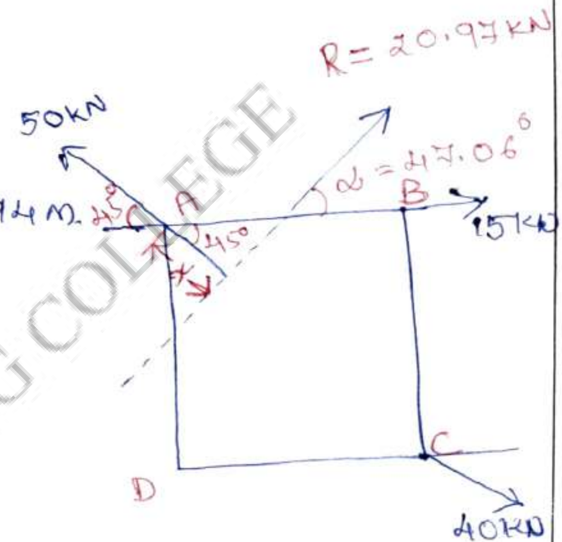


(-) sign shows anticlockwise moment.
Hence moment of resultant force should also be anticlockwise about A. Hence it acts below the point A.

$\Sigma M_A = R \times x.$

$43.92 = 20.97 \times x$

$\therefore x = \dfrac{43.92}{20.97} = 2.094$ m.



Now, $R = 20.97$ KN.
$\alpha = 47.06°$
$x = 2.094$ m.

Suggested Questions / Assignments / Home works / any other

HW. P.T.O.

	Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston Jr., David F Mazurek, Philip J Cornwell, Sanjeev Sanghi,	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press

Any other suggested Materials

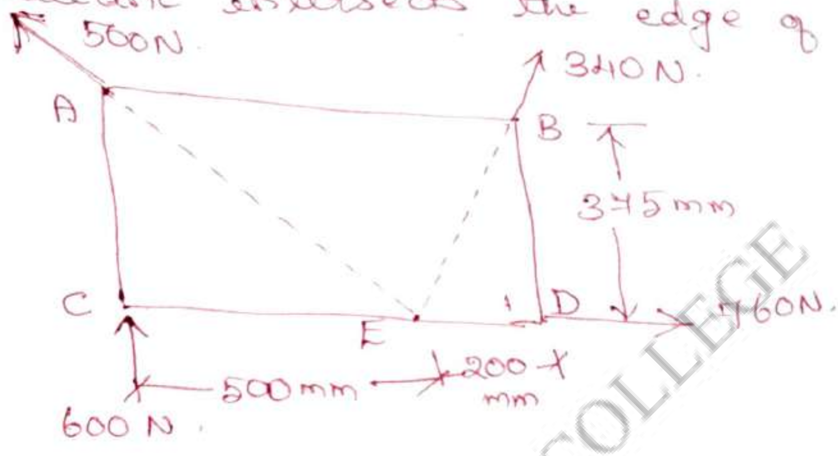
Class notes and Handouts

Home Work:

6. Four fources act on a 700mm x 375 mm plate as shown

a) Find the resultant of these forces.

b) Locate the two points where the line of action of the resultant intersects the edge of the plate.



[Pg. 145, EM by Rottiswaran]

Lecture No. 12

Topic(s) to be covered	Moment of force about an axis.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	apply moment of force about an axis.	L3

Teaching Learning Material	Student Activity
chalk and talk.	Learn and solve.

Lecture Notes

7) A 3000N vertical force is applied to the end of a lever which is attached to a shaft at 'O' as shown. Determine

i) the moment of 3000N force about O.

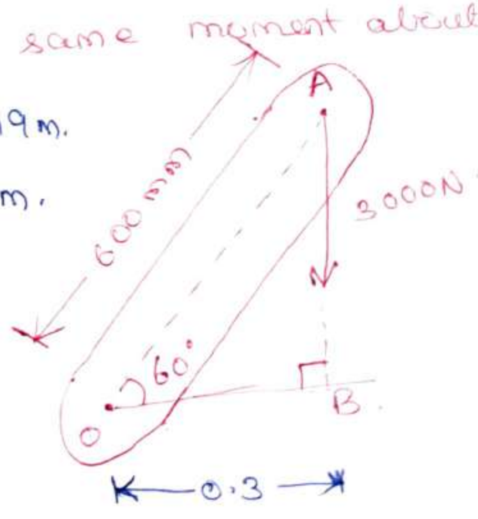
ii) the magnitude of the horizontal force applied at A, which creates the same moment about O.

iii) the smallest force applied at A, which creates the same moment about O.

iv) How far from the shaft a 750N vertical force must act to create the same moment about O.

$AB = 600 \sin 60^\circ = 519.61\,mm = 0.519\,m.$

$OB = 600 \cos 60^\circ = 300\,mm = 0.3\,m.$



i) The moment of 3000N force about O:

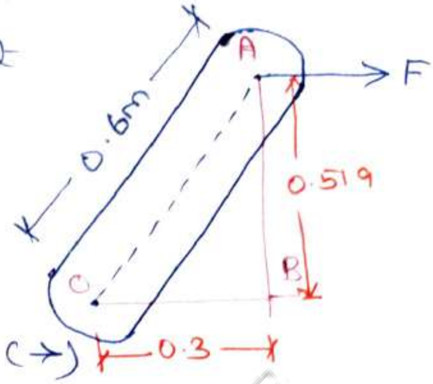
$M_o = 3000 \times OB$

$= 3000 \times 0.3 = 900\,Nm.$

ii) The magnitude of horizontal force applied at A, which creates the same moment about O.

Moment = 900 Nm (clockwise).

Hence horizontal force should act towards right. (to produce cw moment.

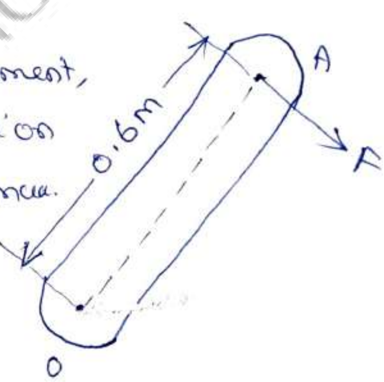


$\therefore 900 = F \times AB$

$$F = \frac{900}{AB} = \frac{900}{0.519} = 1734.1\ N (\rightarrow)$$

iii) The smallest force applied at A, which creates the same moment about O.

For smallest force of the same moment, distance between the line of action of the force and the pt. O is maximum. i.e. 0.6m. Also, the force should be $\perp_r$ to the line OA.



Moment = 900 Nm [cw]

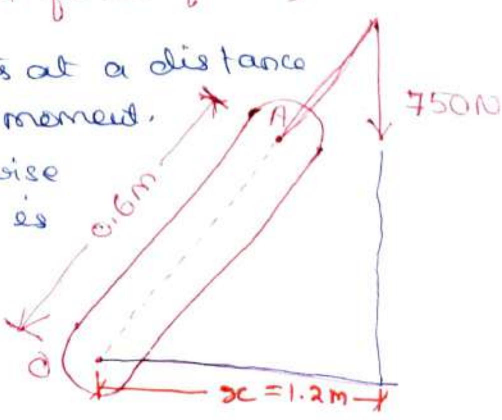
$\therefore$ force should act downwards as shown

$F \times OA = 900$

$$F = \frac{900}{OA} = \frac{900}{0.6} = 1500\ N.$$

iv) Distance of 750N, vertical force from O, to have the same moment.

Let 750 N vertical force acts at a distance of x m from A to have same moment. i.e 900 Nm CW. To have clockwise moment, 750N vertical force is applied on R.H.S. of A.



$750\,x = 900$

$$x = \frac{900}{750} = 1.2\ m.$$

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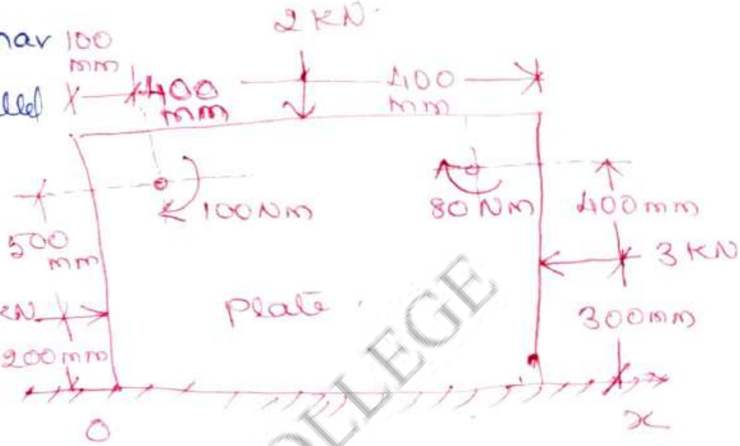
8) A plate is acted upon by 3 forces and 2 couples as shown below. Determine the resultant of these force-couple system and find co-ordinate x of the point on the x-axis through which the resultant passes.

The force system is coplanar non-concurrent and non-parallel forces.



$\Sigma H = 1.5 - 3 = -1.5$ KN. $(\overset{+}{\rightarrow})$

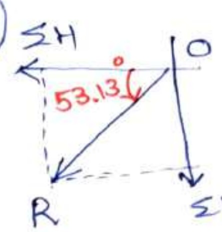
$\Sigma V = -2$ KN. $(\uparrow +)$.

Magnitude of Resultant force

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(-1.5)^2 + (-2)^2} = 2.5 \text{ KN}.$$

Direction of Resultant force, $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$$\alpha = \tan^{-1}\left(\frac{2}{1.5}\right) = 53.13^\circ$$



$R = 2.5$ KN

Location of resultant force:

Algebric sum of moments about O,

$$\Sigma M_0 = (1.5 \times 0.2) - (3 \times 0.3) + (2 \times 0.5) + 0.1 + 0.08$$

$$= 0.3 - 0.9 + 1 + 0.1 + 0.08 = 0.58 \text{ KNm [CW]}$$

From Varignon's theorem,

Algebric sum of moments of all the forces about 'O' = Moment of Resultant force about O.

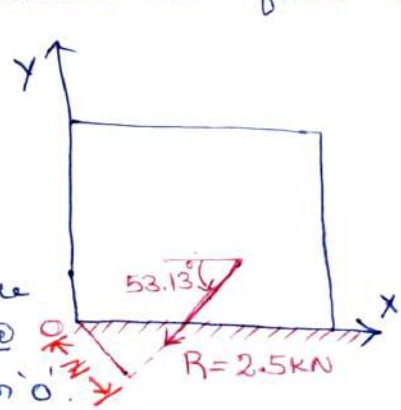
$$\Sigma M_0 = R \times Z$$

$$= (\Sigma V \times x) + (\Sigma H \times 0).$$

$$0.58 = 2 \times x.$$

$\boxed{x = 0.29 \text{ m}}$ (or) 290 mm.

$\therefore$ The resultant force 2.5 KN cuts the x-axis @ a distance of 290 mm from 'O'.



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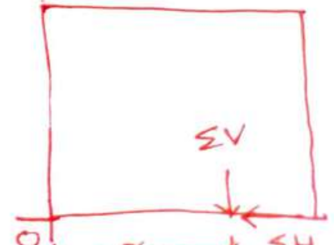


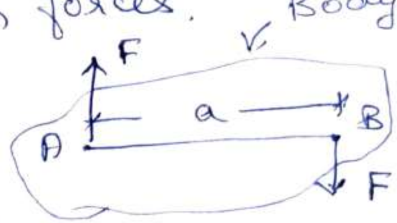
Topic(s) to be covered	Statics of Rigid bodies - Force couple system.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Moment of a couple: The perpendicular distance between the parallel forces is known as arm of the couple. The moment of the couple is the product of either one of the forces and the distance between forces.



Moment of the couple

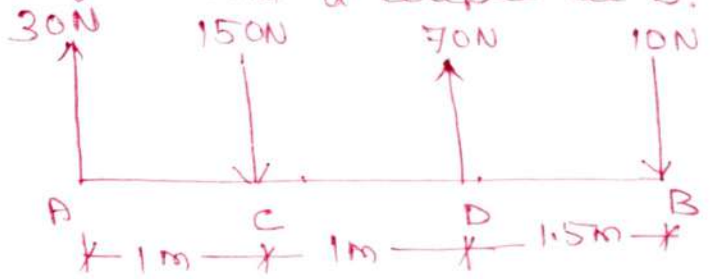
$$M = F \times a$$

Resultant of force couple system:

Note 1: support reactions need not be considered while finding an equivalent force system. Even if support reactions are considered, resultant will be zero.

Note 2: For finding the magnitude of resultant force, magnitude of couple is not considered. It is taken into account only while finding the algebric sum of moments of forces about any point.

(9) A system of parallel forces are acting on rigid bar as shown in fig. Reduce the system to
(i) a single force (ii) a single force and a couple at A
(iii) a single force and a couple at B.



(i) Single force system:

Magnitude of resultant force $R = 30 - 150 + 70 - 10$ ($\uparrow$ +ve)

$$\boxed{R = -60\,N}$$

Direction of resultant force = downward (as R is negative)

Location of resultant force (w.r.t. A).

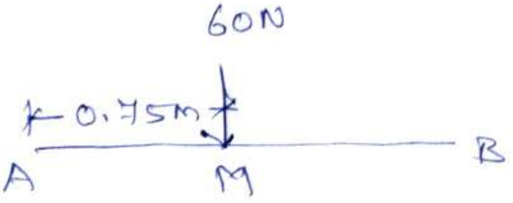
$$R \times x = \Sigma M_A.$$

$$= (30 \times 0) - (150 \times 1) + (70 \times 2) + (10 \times 3.5)$$

$$= 150 - 140 + 35 = 45\ KNm \text{ (clockwise)}$$

$$R x = 45$$

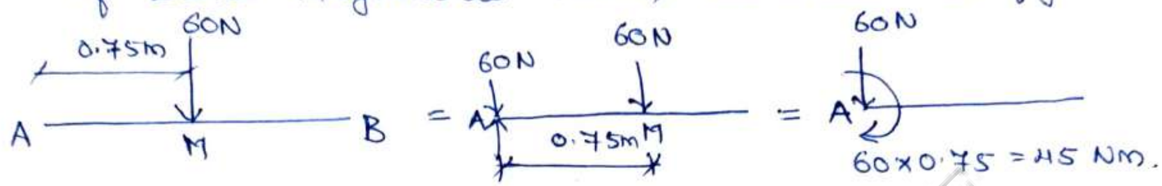
$$x = \frac{45}{60} = 0.75m.$$



Hence the given system of parallel forces is equivalent to a single force 60N acting vertically downward at a distance of 0.75m from A on its R.H.S. as shown.

(ii) A single force and a couple at A.

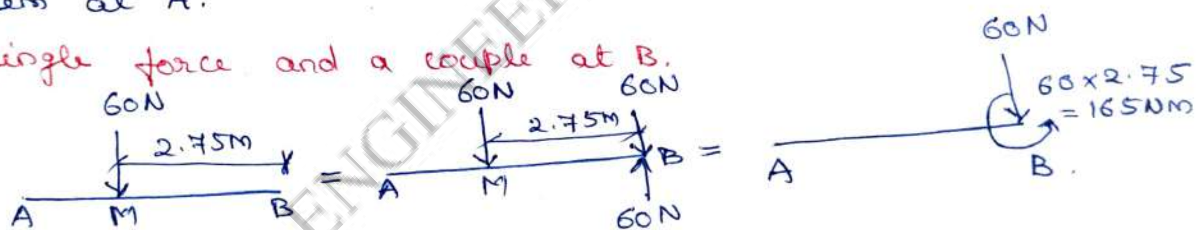
To reduce the resultant force into a force-couple system at A, introduce two unlike collinear forces at A, parallel to the resultant force and of same magnitude (60N) as shown in fig.



Force-couple at A.

The downward force at M and upward force at A form a clockwise couple. The moment of this couple at A is (60×0.75) = 45 Nm and the third force at A (60N downward is unchanged). Fig. shows the force-couple system at A.

(iii) A single force and a couple at B.



Force-couple at B.

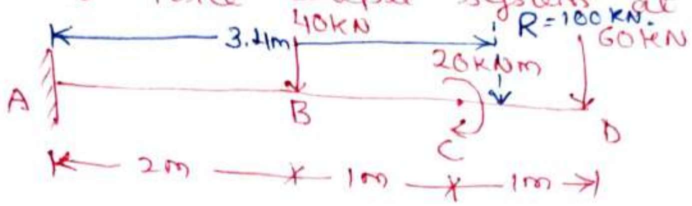
To reduce the resultant force into a force-couple system at B, introduce two unlike collinear force at B, parallel to the resultant force and of same magnitude (60N) as shown in fig.

Now, the downward force at M and upward force at B form an anticlockwise couple. The moment of this couple is (60×2.75) = 165 Nm, and the third force at B (60N downward) is unchanged.

(10). Figure shows two vertical forces and a couple of magnitude 20 KNm, acting on a horizontal rod, which is fixed at A.

(i) Determine the resultant of the system.

(ii) Reduce to force couple system at A.



(i) <u>Resultant of the system</u>:-

magnitude of resultant $= -40 - 60$ $(R\uparrow +ve)$

$= -100$ KN.

Direction of resultant force = downward as R is negative

<u>Location of resultant force</u>:-

Given forces are vertical forces. Hence $\Sigma H = 0$.

$\Sigma M_A = (40 \times 2) + (60 \times 4) + 20$

$= 80 + 240 + 20$

$= 340$ KNm. (clockwise).

From Varignon's theorem,

Algebric sum of moments about A = moment of resultant force about A.

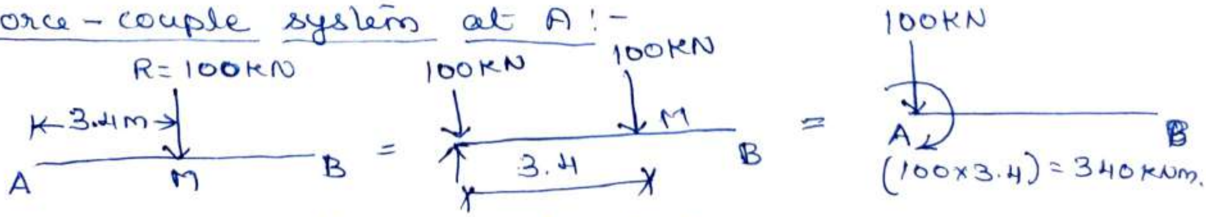
$\Sigma M_A = R \times x$.

$340 = 100 \times x$.

$\therefore x = \frac{340}{100} = 3.4$ m.

Resultant force for the given system is 100 KN, acting vertically downward at a distance of 3.4 M R.H.S. of A.

(ii) <u>Force-couple system at A</u>:-



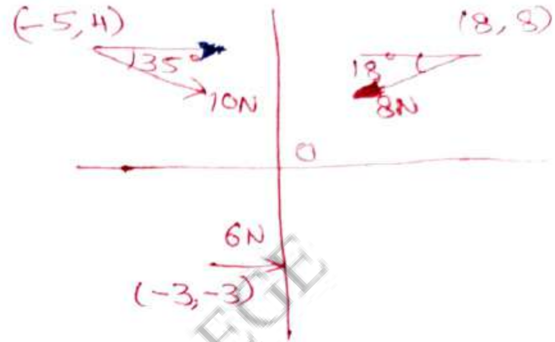
Force - couple at A.

(11) For the force system shown in figure,

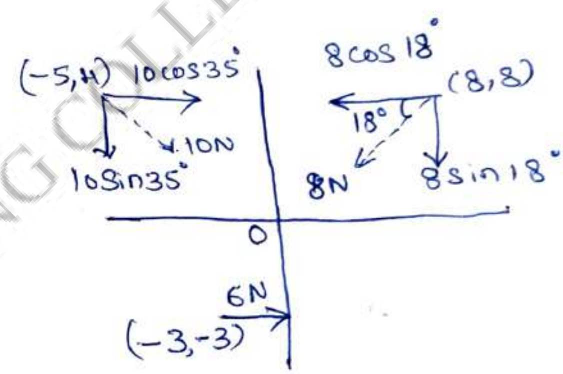
(i) Determine the resultant force.

(ii) Reduce to an equivalent force couple system at origin O.

Assume the co-ordinates are in m.



The given forces are resolved into two components in x and y directions as shown.



(i) Resultant force

$(\rightarrow +)\ \Sigma H = 10\cos 35° - 8\cos 18° + 6$

$= 8.191 - 7.608 + 6$

$= 6.583\ N.$

$(\uparrow +)\ \Sigma V = -10\sin 35° - 8\sin 18° + 0$

$= -5.735 - 2.472 + 0$

$= -8.207\ N.$

Magnitude of resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(6.583)^2 + (8.207)^2}$$

$$= 10.52\ N.$$

Direction of resultant force:

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{8.207}{6.583}\right) = 51.26°$$

Location of resultant force:

Let the ⊥r distance of resultant force from origin be 'x' m.

Algebric sum of moments of all forces about origin,

$(\circlearrowleft +)\ \Sigma M_0 = (10\cos 35° \times 4) + (8\sin 18° \times 8) - (10\sin 35° \times 5) - (8\cos 18 \times 8) - (6 \times 3)$

$= 32.76 + 19.78 - 28.68 - 60.86 - 18$

$= -55\ KNm.$

$$\boxed{\Sigma M_o = -55\ \text{KNm}}$$

Negative sign shows anticlockwise moment. Hence resultant force should also produce " " about origin. To satisfy the direction of resultant force and the nature of anticlockwise moment, it must be taken on the leftside of the origin.

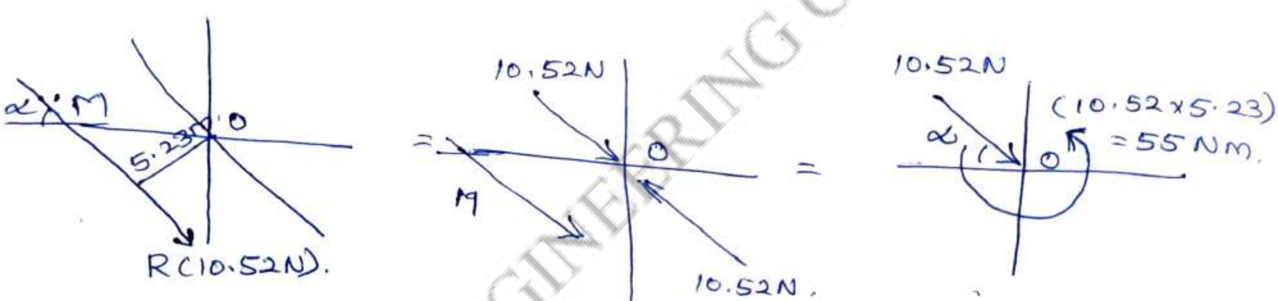
From Varignon's theorem,

Sum of moments about origin = moment of resultant force about origin.

$$\Sigma M_o = R \times x$$

$$\therefore x = \frac{\Sigma M_o}{R} = \frac{55}{10.52} = 5.23\ m.$$

$$\boxed{\alpha = 51.26^\circ}$$



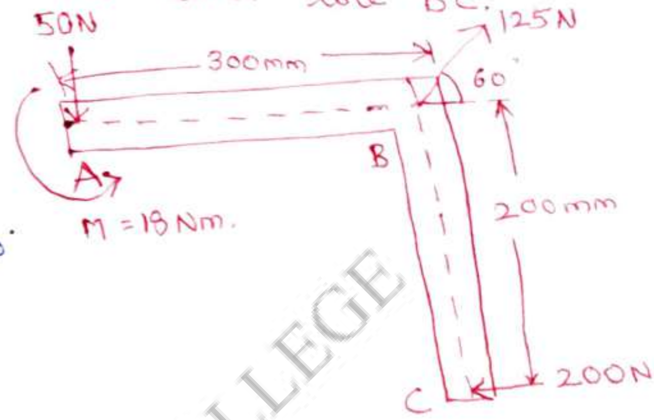
① Force-couple system at origin. ② ③

Resultant force is shown in fig ①, to reduce this into a force-couple system at origin, apply two equal and opposite collinear forces at origin, parallel to the resultant force and of same magnitude. Now the downward force at M and upward force at 'o' form an anticlockwise couple of $(10.52 \times 5.23) = 55.01\ Nm$. The downward force at origin is unchanged. Figure shows the force-couple system at origin.

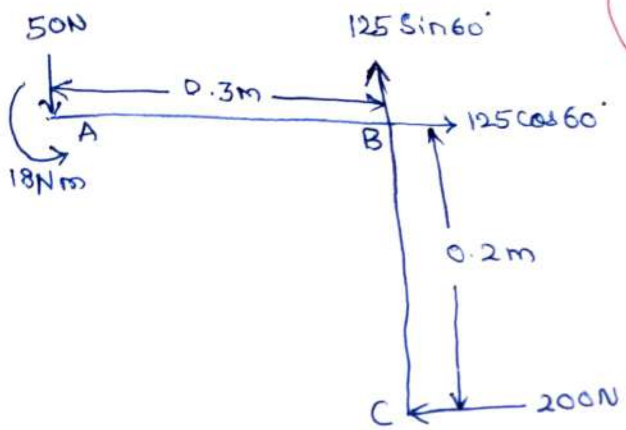
(12). The three forces and a couple of magnitude, M = 18 Nm are applied to an angled bracket as shown in fig.

(a) Find the resultant of this system of forces

(b) Locate the points where the line of action of the resultant intersects line AB and line BC.



Solution:

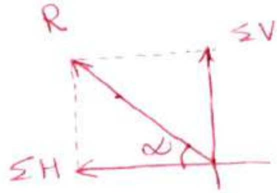


(i) Resultant force: Algebraic sum of horizontal forces,

$\Sigma H = 125\cos 60° - 200 = -137.5 N.$

Algebraic sum of vertical forces,

$\Sigma V = 125 \sin 60° - 50 = 58.25 N.$



Magnitude of resultant forces,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(137.5)^2 + (58.25)^2} = 149.32 N.$$

Direction of resultant force, $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$$= \tan^{-1}\left(\frac{58.25}{137.5}\right) = 22.95°.$$

Note: What ever be the magnitude and nature of couple M, it will not affect the magnitude and direction of resultant force.

Location of Resultant force: Let us locate resultant force w.r.t. pt. A., $\Sigma M_A = (200 \times 0.2) - (125 \sin 60° \times 0.3) - 18.$

$= 40 - 32.475 - 18$

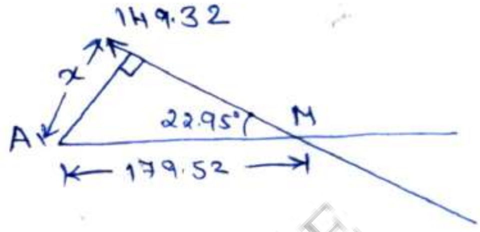
$= -10.475$ Nm. (-ve Sign shows anticlockwise moment)

Let the ⊥r. distance of resultant force from A be 'x' m.

Applying Varignon's theorem,

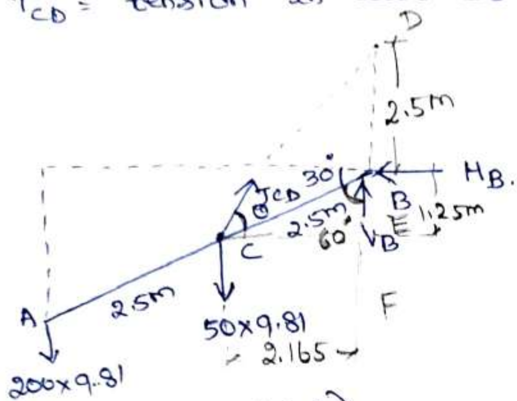
$$\Sigma M_A = R \times x.$$

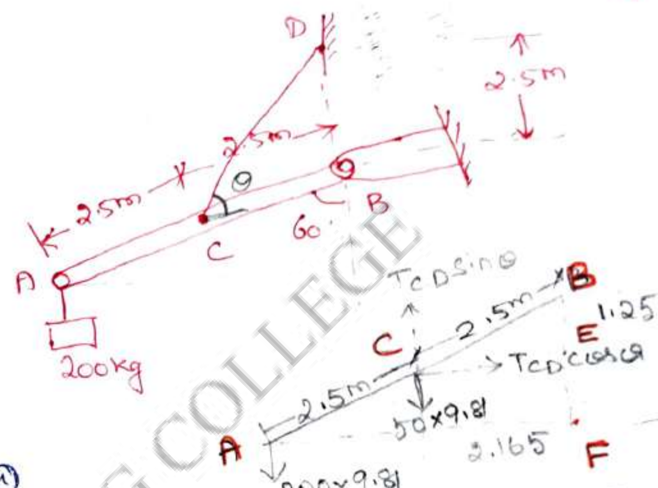
$$\therefore x = \frac{\Sigma M_A}{R} = \frac{10.475 \text{ Nm}}{149.32 \text{ N}} = 0.07 \text{m} = 70 \text{mm}.$$



13) A uniform bar AB shown in fig. has a mass of 50 kg and supports a mass of 200 kg at A. A supporting cable is tied to the bar at C and the other end is fixed to the vertical wall at D. Calculate the tension in the supporting cable and the magnitude of reaction force at pin B.

Let T_{CD} = tension in cable CD.



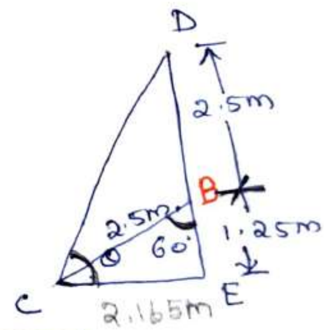


applying $\Sigma H = 0$, $(\rightarrow +)$.

$$T_{CD}\cos\theta - H_B = 0 \quad \text{--- (1)}$$

applying $\Sigma V = 0 \Rightarrow T_{CD}\sin\theta + V_B - (200\times 9.81) - (50\times 9.81) = 0.$ --- (2)

where θ = angle of T_{CD} with horizontal.



To find θ, In Δ^{le} BCE,

$$\sin 60° = \frac{CE}{2.5} = \frac{opp}{hyp}$$

$$CE = 2.5\sin 60° = 2.165\,m.$$

lly $\cos 60° = \frac{BE}{2.5}$

$$BE = 2.5\cos 60° = 1.25\,m.$$

In Δ^{le} CDE, $\theta = \tan^{-1}\left(\frac{2.5+1.25}{2.165}\right) = 60° \Rightarrow \boxed{\theta = 60°}$

Applying $\Sigma M_B = 0$ $(\circlearrowright +)$.

$$(T_{CD}\sin\theta \times CE) - (T_{CD}\cos\theta \times BE) - (50\times 9.81\times CE) - (200\times 9.81\times 5\sin 60) = 0.$$

(AF)

$$(T_{CD}\sin\theta \times CE) - (T_{CD}\cos\theta \times BE) - (50\times 9.81\times 2.165) - (200\times 9.81\times 5\sin 60) = 0$$

$$\Rightarrow (T_{CD}\sin 60° \times 2.165) - (T_{CD}\cos 60 \times 1.25) - (50\times 9.81\times 2.165) - (200\times 9.81\times 5\sin 60) = 0$$

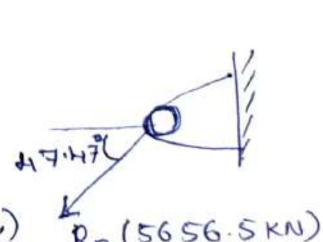
$$\sin 60 = \frac{AF}{AB} = \frac{AF}{5m}$$

$$\Rightarrow (T_{CD}\sin 60° \times 2.165) - (T_{CD}\cos 60 \times 1.25) - 1061.93 - 8495.7 = 0.$$

$$\Rightarrow 1.875\,T_{CD} - 0.625\,T_{CD} = 9557.63\,N.$$

$$1.25\,T_{CD} = 9557.63\,N.$$

$$\therefore \boxed{T_{CD} = 7646\,N.}$$



Sub. T_{CD} and θ in equ. (1) & (2),

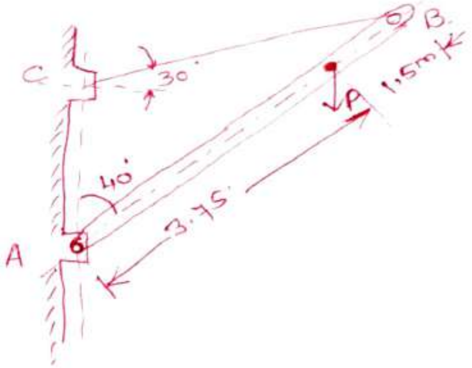
$H_B = 3823\,N\ (\leftarrow)$; $V_B = -4169\,N = 4169\,N\ (\downarrow)$

$R_B = \sqrt{V_B^2 + H_B^2} = 5656\,N.$; $\alpha = \tan^{-1}\left(\frac{V_B}{H_B}\right) = \tan^{-1}\left(\frac{4169}{3823}\right) = 47.47°.$

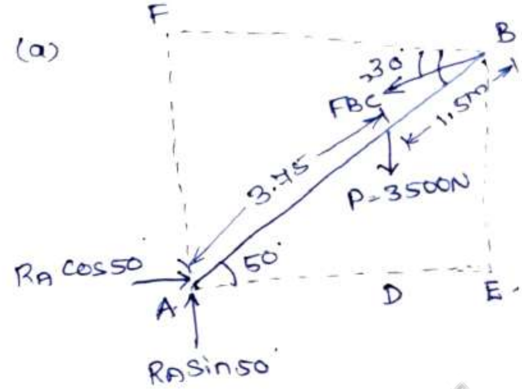
(14) A load P of 3500 N is acting on the boom, which is held by a cable BC as shown. The wt. of the boom can be neglected.

a) Draw the FBD of boom b) Find out the tension in cable BC

c) Determine the reaction at A.



(a)



Free Body Diagram of the boom.

(b) Tension in the cable BC:

Applying $\Sigma M_A = 0$ (↺+).

$(3500 \times AD) + (F_{BC} \sin 30' \times AE) - (F_{BC} \cos 30 \times BE) = 0.$

From Geometry, $AD = 3.75 \cos 50' = 2.410\,m.$

$AE = 5.25 \cos 50' = 3.374\,m.$

$AF = BE = 5.25 \sin 50' = 4.022\,m.$

$\therefore (3500 \times 2.41) + (F_{BC} \sin 30 \times 3.374) - (F_{BC} \cos 30' \times 4.022) = 0.$

$8435 + 1.687 F_{BC} - 3.483 F_{BC} = 0$

$8435 - 1.796 F_{BC} = 0$

$\therefore \boxed{F_{BC} = 4696.5\,N}$

(c) Reaction at A:

$\Sigma H = 0$ (→+) $\Rightarrow H_A - F_{BC} \cos 30' = 0.$

$H_A = F_{BC} \cos 30' = 4696.5 \cos 30' = 4067.2\,N.$

$\Sigma V = 0$ (↑+) $\Rightarrow V_A - 3500 - F_{BC} \sin 30' = 0.$

$V_A - 3500 - (4696.5 \sin 30') = 0.$

$V_A - 3500$

$\boxed{V_A = 5848.25\,N}$

$R = \sqrt{V_A^2 + H_A^2} = \sqrt{(4067.2)^2 + (5848.2)^2}$

$\boxed{R = 7123.45\,N}$

$\alpha = \tan^{-1}\left(\frac{5848.25}{4067.2}\right)$

$\boxed{\alpha = 55.18°}$

Lecture No. 16, 17 **UNIT II – EQUILIBRIUM OF RIGID BODIES**

Topic(s) to be covered	Support reactions on beam.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	Understand different types of support	
LO2	find reactions of a beam.	

Teaching Learning Material	Student Activity
chalk and Talk.	Learn and solve.

Lecture Notes

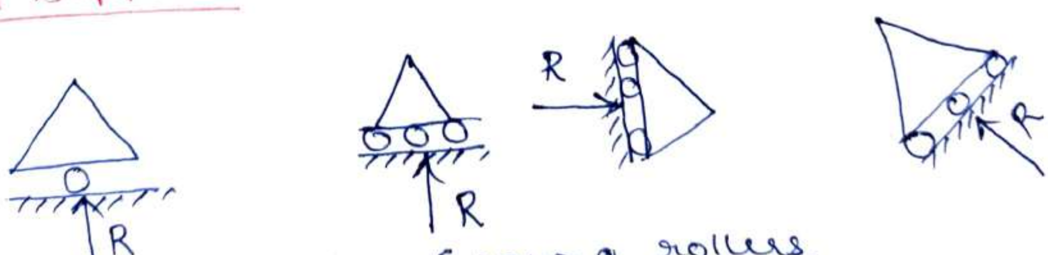
Support reactions of a beam:

The force of resistance exerted by the support on the beam is called as support reaction. Support reaction of beam depends upon the type of loading and the type of support.

Types of support:

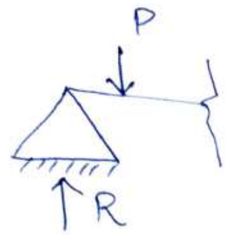
1. Roller support 2. Hinged support 3. Fixed support.

1. Roller support:

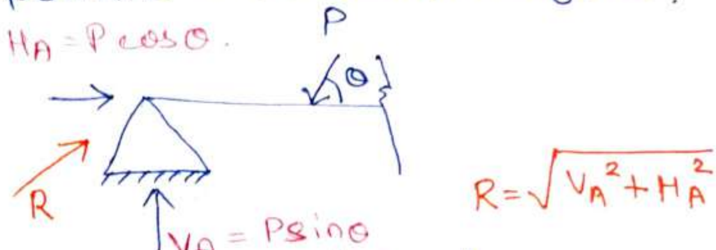


a) Single roller b) Group of rollers.

2. Hinged Support: This type of support can withstand any type of force both horizontal and vertical. Hence, it has 2 reaction components – vertical & horizontal.



a) vertical load.



b) Inclined load.

Hinged support is also called pin-jointed support.

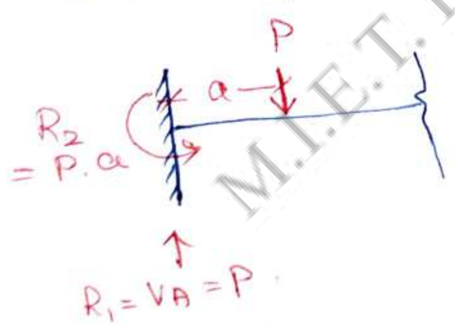
3. Fixed Support: Both roller and hinged supports can resist only displacements (i.e. vertical and horizontal movement of beam at ends, but rotation of beam is not resisted by both the supports. Fixed support can resist rotation.

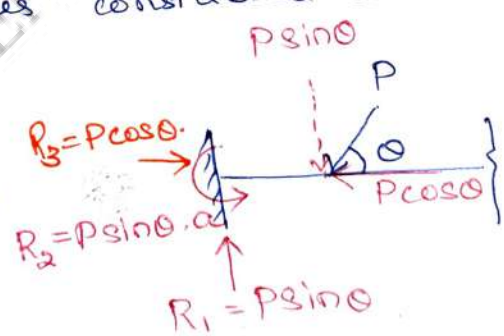
Fixed support has 3 reaction components. i) Horizontal reaction.

ii) vertical reaction.

iii) rotational reaction.

Fixed support is considered as strongest support.



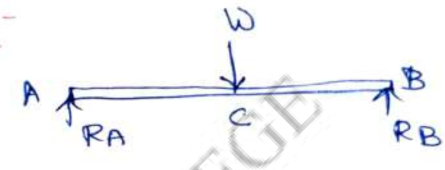


or built-in support:

In case of fixed support, the reaction will be inclined. Also the fixed support will provide a couple.

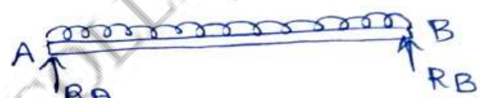
TYPES OF LOADING

(a) concentrated or point load:-



Any load acting at a point on a beam is called ~.

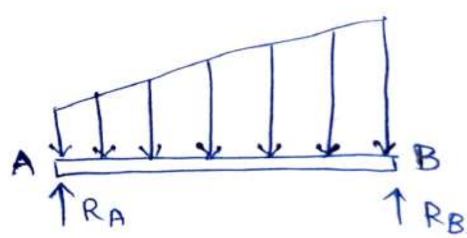
(b) uniformly distributed load:-



If a beam is loaded in such a way that each unit length of the beam carries same intensity of the load, then that type of load is called ~.

For finding the reactions, the total udl is assumed to act at the C.G. of the load.

(c) uniformly varying load:-



The rate of loading on each unit length of the beam varies uniformly,

total load = area of load diagram.

total load acts at the C.G. of the load diagram.

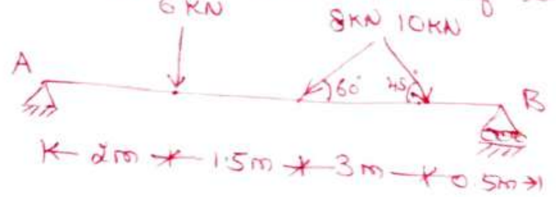
Moments and couples:-

Moment:- is the product of the force and the perpendicular distance between the line of action of the force and the pt about which moment is to be taken.

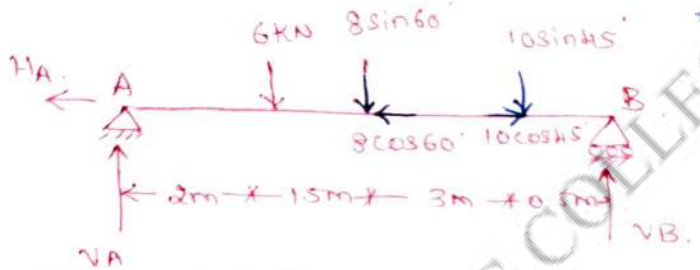
couple:- when two equal and opposite parallel forces act on a body at some distance apart, the two forces form a couple. This couple has a tendency to rotate the body.

Finding the support reactions.

1. Determine the support reactions of the beam shown.



First of all, the inclined forces are to be resolved into two components in vertical and horizontal directions.



* Applying $\Sigma V = 0 \ (\uparrow +)$

$V_A + V_B - 6 - 8\sin 60° - 10\sin 45° = 0$

$\therefore V_A + V_B = 20 \text{ KN} \quad \ldots (1)$

Applying $\Sigma H = 0 \ (\rightarrow +)$.

$10\cos 45° - 8\cos 60° - H_A = 0$.

$H_A = 10\cos 45° + 8\cos 60° = 3.07 \text{ KN}$.

H_A is +ve and hence direction of H_A assumed is correct ($\leftarrow$)

* Applying $\Sigma M_A = 0$ (↻+)

$(6 \times 2) + (8\sin 60° \times 3.5) + (10\sin 45° \times 6.5) - (V_B \times 7) = 0$.

$V_B \times 7 = 82.2$. $\quad \therefore \boxed{V_B = 11.74 \text{ KN}}$

Sub. in (1),

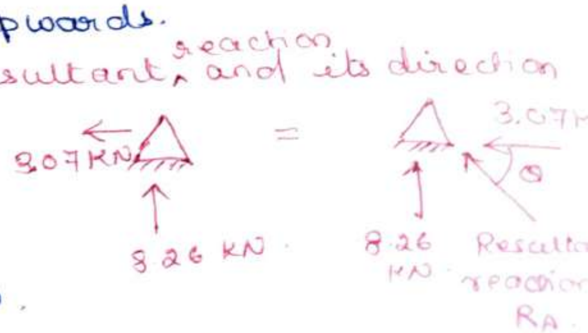
$V_A + V_B = 20$

$V_A + 11.74 = 20$

$\boxed{V_A = 8.26 \text{ KN}}$

Both V_A and V_B are positive, hence assumed directions are correct. Both are acting upwards.

In the above problem, if the resultant reaction and its direction at hinged support is reqd,

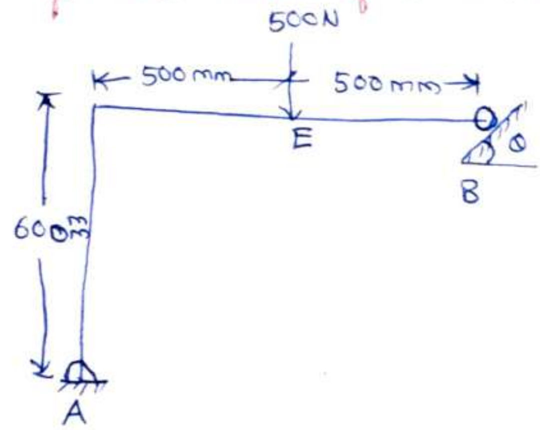


$R_A = \sqrt{(V_A)^2 + (H_A)^2}$

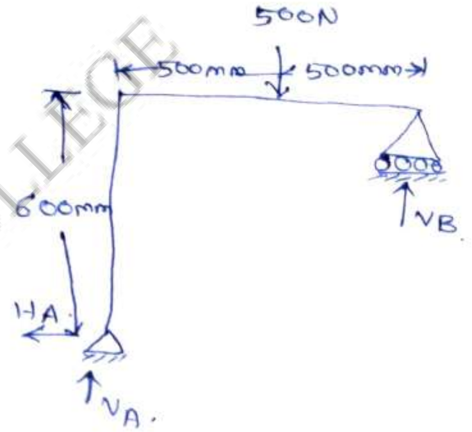
$= \sqrt{(8.26)^2 + (3.077)^2} = 8.81 \text{ KN}$.

and $\theta = \tan^{-1}\left(\frac{V_A}{H_A}\right) = \tan^{-1}\left(\frac{8.26}{3.07}\right) = 69.61°$.

A frame supported at A and B is subjected to force of 500N as shown below. Compute the reactions at support points for the cases of $\theta = 0°$, $\theta = 90°$ and $\theta = 60°$ (Anna, Nov/Dec 2003)



(i) $\boxed{\theta = 0°}$



Applying $\Sigma H = 0$ $(\rightarrow +)$.

$-H_A = 0$

Applying $\Sigma V = 0$ $(\uparrow +)$.

$V_A + V_B - 500 = 0$

$V_A + V_B = 500 \quad \ldots (1)$

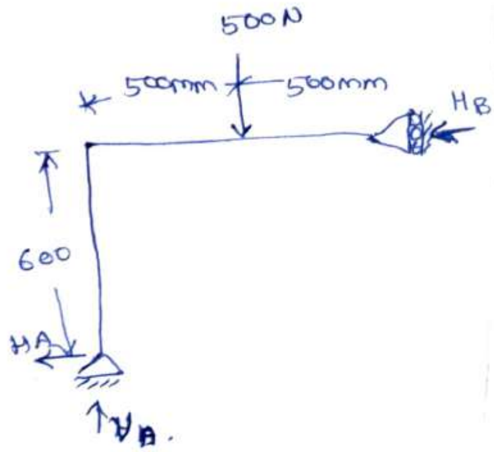
Applying $\Sigma M_A = 0$ $(\circlearrowright +)$

$(500 \times 500) - (V_B \times 1000) = 0$

$V_B = 250N$.

Sub. in (1), $\therefore V_A = 500 - 250$
$= 250N$.

(ii) $\boxed{\text{when } \theta = 90°}$



Applying $\Sigma H = 0$ $(\rightarrow +)$,

$-H_A - H_B = 0$.

Applying $\Sigma V = 0$ $(\uparrow +)$.

$V_A - 500 = 0$

$\therefore \boxed{V_A = 500N}$

Applying $\Sigma M_A = 0$ $(\circlearrowright +)$.

$(500 \times 500) - (H_B \times 600) = 0$

$\boxed{H_B = 416.67 \text{ N}}$

$-H_A - H_B = 0$

$-H_A = 416.67 \text{ N}$

$\boxed{H_A = -416.67 \text{N}}$

(iii) $\boxed{\text{when } \theta = 60^\circ}$

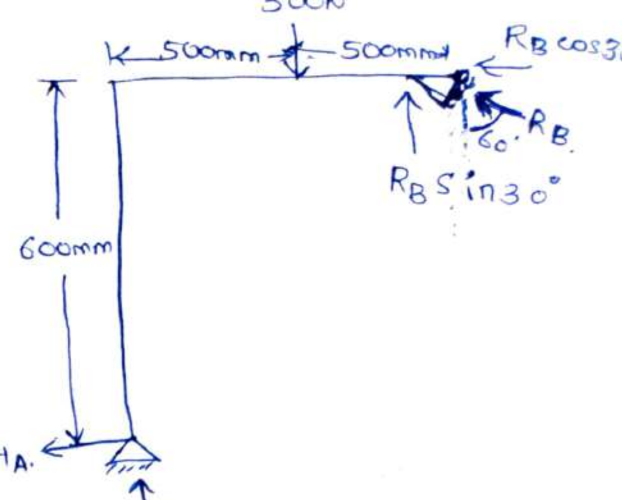
Applying $\Sigma H = 0 \ (\rightarrow +)$

$$-H_A - R_B \cos 30^\circ = 0 \ \ldots\ldots (1)$$

Applying $\Sigma V = 0 \ (\uparrow +)$

$$V_A - 500 + R_B \sin 30^\circ = 0$$

$$V_A + 0.5 R_B = 500 \ \ldots (2)$$



Applying $\Sigma M_A = 0 \ (\circlearrowright +)$

$$(500 \times 500) - (R_B \sin 30^\circ \times 1000) - R_B (\cos 30^\circ \times 600) = 0.$$

$$250{,}000 - 500 R_B - 519.61 R_B = 0.$$

$$250000 = 1019.61 R_B.$$

$$\therefore \boxed{R_B = 245.19\,N}$$

Sub. R_B in equ. (1),

$$-H_A - (245.19 \cos 30^\circ) = 0.$$

$$H_A = -212.34 N \ (\leftarrow)$$

$$\therefore \boxed{H_A = 212.34 N \ (\rightarrow)}$$

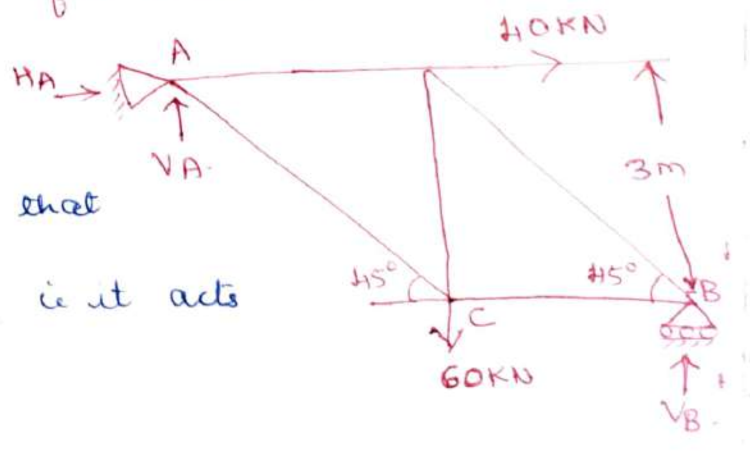
Sub. R_B in equ. (2)

~~VA + 500 + (245.19 sin 30) = 0~~

$$V_A + (0.5 \times 245.19) = 500.$$

$$\therefore \boxed{V_A = 377.4 N \ (\uparrow)}$$

(3) Find the support reactions of a truss shown.



Applying $\Sigma H = 0, \ (\rightarrow +)$

$$H_A + 40 = 0.$$

$$H_A = -40$$ (-ve sign indicates that the assumed direction is wrong ie it acts in opp. direction.

Applying $\Sigma V_A = 0. \ (\uparrow +)$

$$V_A + V_B - 60 = 0.$$

$$V_A + V_B = 60 \ \ldots\ldots (1)$$

Applying $\Sigma M_A = 0$ (↺+). ($CB = AD = 3\tan 45^\circ = 3m$)

$(60 \times 3) - (V_B \times 6) = 0$

$(60 \times 3) = 6V_B$

$\therefore \boxed{V_B = 30\ KN}$

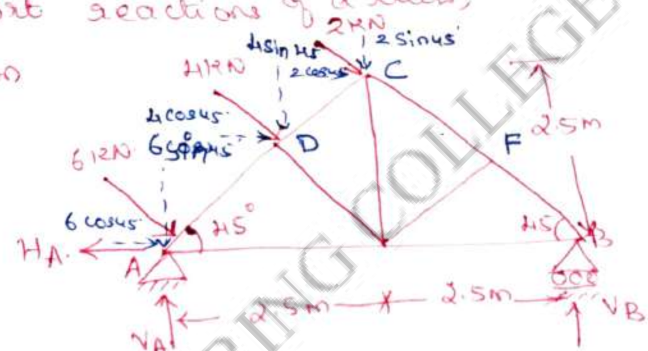
Sub. $V_B = 30\ KN$ in equ. (1), we get $\boxed{V_A = 30\ KN}$

Result: Horizontal reaction at A = 40 KN (←)

Vertical reaction at A = 30 KN (↑)

Vertical reaction at B = 30 KN (↑).

(4) Find the support reactions of a truss, loaded as shown



$\Sigma H = 0$. (→+).

$6\cos 45^\circ + 4\cos 45^\circ + 2\cos 45^\circ - H_A = 0$.

$H_A = 12\cos 45^\circ = 8.485\ KN$ (←).

$\Sigma V = 0$. (↑+)

$V_A + V_B - 6\sin 45 - 4\sin 45^\circ - 2\sin 45^\circ = 0$.

$\therefore V_A + V_B = 12\sin 45^\circ = 8.485\ KN$.

$\Sigma M_A = 0$. (↺+),

$(4 \times AB) + 2(AC) - (V_B \times AB) = 0$. .... (1)

$AD = 2.5\cos 45^\circ = 1.767\,m$.

$AC = 5\cos 45^\circ = 3.534\,m$,

Sub. in (1),

$(4 \times 1.767) + (2 \times 3.534) - (V_B \times 5) = 0$.

$5V_B = 14.136$.

$\therefore \boxed{V_B = 2.827\ KN}$ ↑

$\therefore \boxed{V_A = 5.658\ KN}$ ↑

$\boxed{H_A = 8.485\ KN}$ ←

Lecture No. 18 **UNIT II – EQUILIBRIUM OF RIGID BODIES**

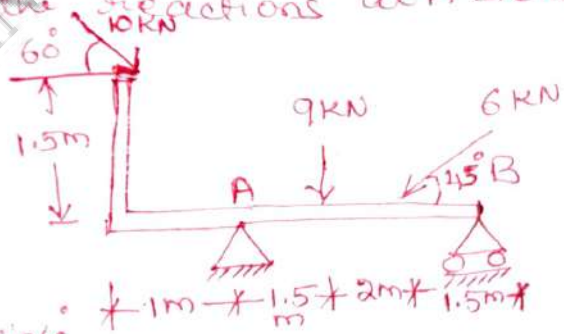
Topic(s) to be covered	Reaction at support.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	find the support reactions	K3

Teaching Learning Material	Student Activity
chalk and Talk	Learn and solve

Lecture Notes

(5) A beam AB is simply supported and carries load as shown in fig. Calculate the reactions at A and B.



Applying $\Sigma H = 0$ ($\rightarrow$ +ve)

$10\cos 60° - 6\cos 45° - H_A = 0$

$\therefore H_A = 10\cos 60° - 6\cos 45°$

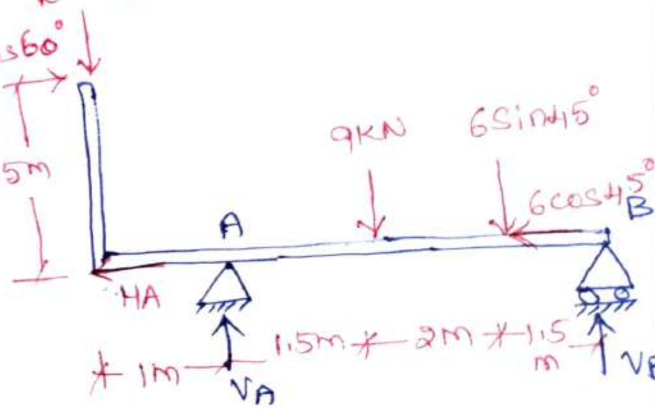
$H_A = 5 - 4.24 = 0.76$ kN.

Applying $\Sigma V = 0$ ($\uparrow$ +ve)

$V_A + V_B - 10\sin 60 - 9 - 6\sin 45 = 0.$

$\therefore V_A + V_B = 8.66 + 9 + 4.24$

$= 21.9$ kN. ——— (1)



Applying $\Sigma M_A = 0$ (↺ +ve)

$(9 \times 1.5) + (6\sin 45 \times 3.5) +$

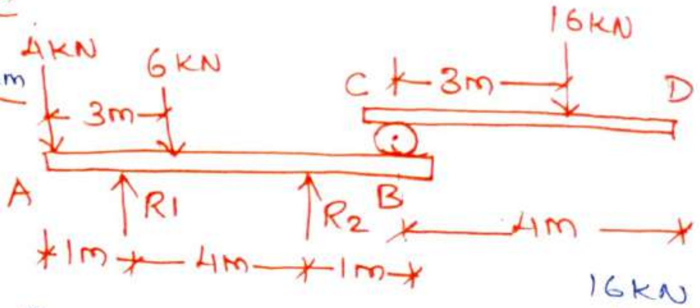
$(10\cos 60 + 1.5) - (10\sin 60 \times 1) - (V_B \times 5) = 0.$

$\Rightarrow 5V_B = 13.5 + 14.85 + 7.5 - 8.66 = 27.19.$

$\boxed{V_B = 5.438 \text{ kN}}$

sub. in equ. (1), $\boxed{V_A = 16.462 \text{ kN}}$ $\boxed{H_A = 0.76 \text{ kN}}$

6. Calculate the reactions R_1, R_2 and R_3 for the two beams AB and CD supported as shown. There being a hinge connecting B and C.



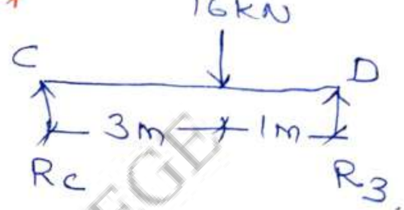
Considering equilibrium of beam CD,

Hinge @ C offers a vertical reaction R_c.

$\Sigma V = 0$, $R_c + R_3 - 16 = 0$.

$R_c + R_3 = 16$. ---(1).

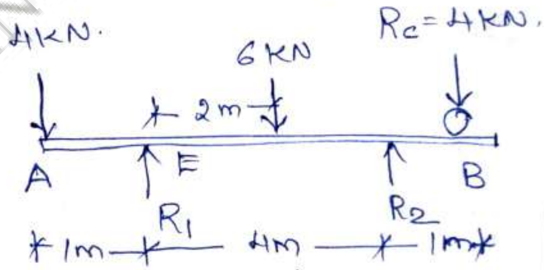
$\Sigma M_c = 0$, $(16\times 3) - (R_3 \times 4) = 0$.



$R_3 = \dfrac{16\times 3}{4} = 12$ kN. Sub. in (1), $R_c = 4$ kN.

considering equilibrium of beam AB,

R_c of beam CD, exerts downward load at B in beam AB



Applying $\Sigma V = 0$,

$R_1 + R_2 - 4 - 6 - 4 = 0$.

$R_1 + R_2 = 14$ ...... (2)

Applying $\Sigma M_B = 0$

$(6\times 2) + (4\times 5) - (4\times 1) - (R_2 \times 4) = 0$

$4R_2 = 12 + 20 - 4 = 28$ kN.

$\boxed{R_2 = 7 \text{ kN}}$

Sub. in equ. (2)

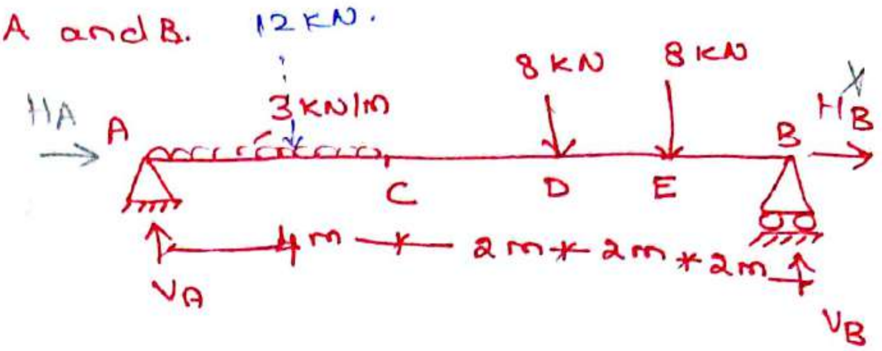
$\boxed{R_1 = 7 \text{ kN}}$

$\boxed{R_1 = 7 \text{ kN} (\uparrow)}$

$\boxed{R_2 = 7 \text{ kN} (\uparrow)}$

$\boxed{R_3 = 12 \text{ kN} (\uparrow)}$

(7) A beam of span 10m is loaded as shown. Determine the reactions at A and B.



Applying $\Sigma H = 0$

$$\boxed{H_A = 0}$$

Applying $\Sigma V = 0$.

$$V_A + V_B - 8 - 8 - (3 \times 4) = 0.$$

$$\boxed{V_A + V_B = 28 \text{ KN.}}$$

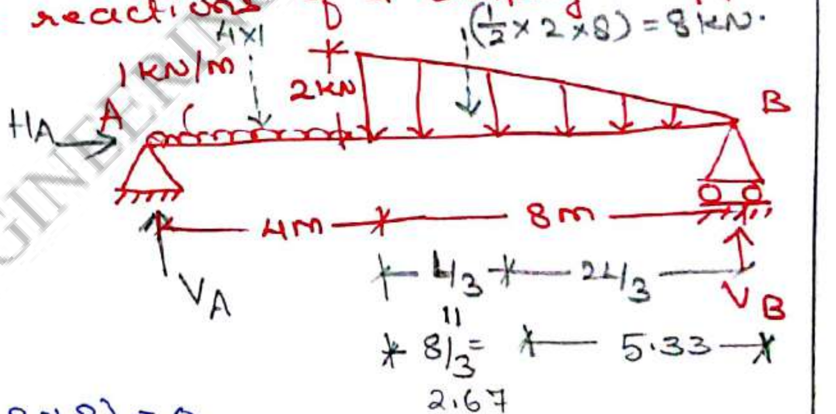
Applying $\Sigma M_A = 0$.

$$(12 \times 2) + (8 \times 6) + (8 \times 8) - (V_B \times 10) = 0$$

$$\boxed{V_B = 13.6 \text{ KN}}$$

$$\boxed{V_A = 14.4 \text{ KN}}$$

(8) Calculate the support reactions of a simply supported beam shown below:



Applying $\Sigma H = 0$,

$$\boxed{H_A = 0}$$

Applying $\Sigma V = 0$,

$$V_A + V_B - (1 \times 4) - \left(\tfrac{1}{2} \times 2 \times 8\right) = 0$$

$$\boxed{V_A + V_B = 12 \text{ KN}} \quad \ldots\ldots (1)$$

Applying $\Sigma M_A = 0$,

$$(4 \times 2) + [8 \times (4 + 2.67)] - (V_B \times 12) = 0$$

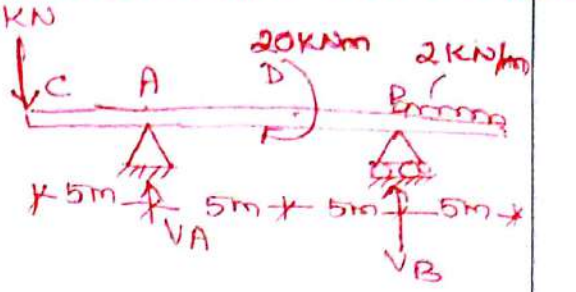
$$12 V_B = 8 + 53.36$$

$$\boxed{V_B = 5.11 \text{ KN} (\uparrow)}$$

Sub. in equ. (1),

$$\boxed{V_A = 6.89 \text{ KN} (\uparrow)}$$

(9) A S/S overhanging beam 20m long carries a system of loads and a couple as shown. Determine the reactions at support A and B.



Applying $\Sigma H = 0$, $\Rightarrow$ $\boxed{H_A = 0}$

Applying $\Sigma V = 0$,

$$V_A + V_B - 8 - (2 \times 5) = 0$$

$$\therefore \boxed{V_A + V_B = 18\,kN}$$

Applying $\Sigma M_A = 0$

$$(2 \times 5 \times 12.5) - (8 \times 5) + 20 - (V_B \times 10) = 0$$

$$\boxed{V_B = 10.5\,kN}$$

$$\boxed{V_A = 7.5\,kN}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston Jr., David F Mazurek, Philip J Cornwell, Sanjeev Sanghi,	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press

Any other suggested Materials

Class notes and Handouts

Lecture No. 19 **UNIT III – DISTRIBUTED FORCES**

Topic(s) to be covered	Centroid and Centre of gravity.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1 LO2	understand centroid & centre of gravity. understand axis of symmetry.	L2 L2

Teaching Learning Material	Student Activity
chalk and Talk	learn and write.

Lecture Notes

Introduction: surfaces are two dimensional whereas solids are three dimensional bodies.

For 2-dimensional bodies – area is to be determined.

3-dimensional bodies – volume is to be determined.

Centroid and Centre of gravity:

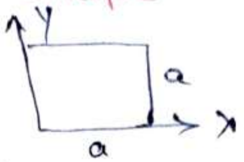
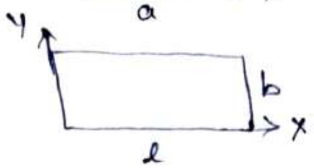
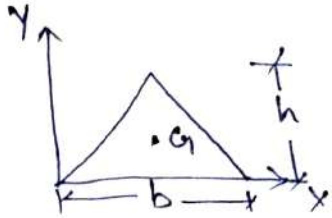
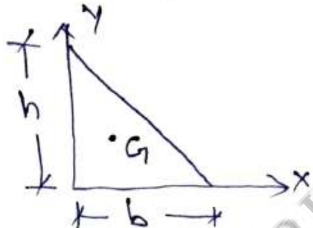
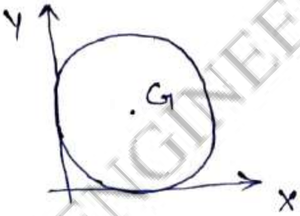
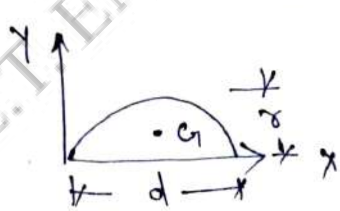
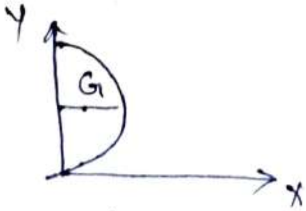
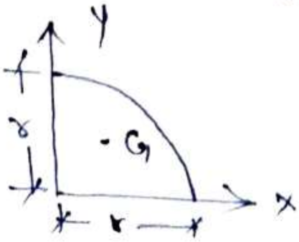
The centre of gravity of a body is defined as the point through which the entire wt. of the body acts; when this is referred to weightless laminas or plane areas (plane areas have no mass) is called the centroid of the area

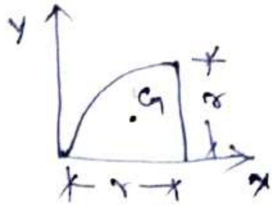
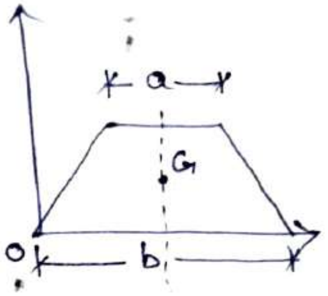
or

centroid is the term referred to one and two dimensional figures and centre of gravity is referred to 3 dimensional figures, Both are represented by 'G'

Chapter 12, Pg. 402

Centroid of simple plane figures:

S.No.	Name	Shape	$\bar{x}$	$\bar{y}$	Area
1.	Square		$\frac{a}{2}$	$\frac{a}{2}$	a^2
2.	Rectangle		$\frac{l}{2}$	$\frac{b}{2}$	$l \times b$.
3.	Triangle (Isosceles)		$\frac{b}{2}$	$\frac{h}{3}$	$\frac{1}{2} bh$.
4.	Triangle (Right-angled)		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2} bh$
5.	Circle.		$\frac{d}{2}$	$\frac{d}{2}$	$\frac{\pi d^2}{4}$
6.	Semi-circle.		$\frac{d}{2}$	$\frac{4r}{3\pi}$	$\frac{1}{2} \times \frac{\pi d^2}{4}$
7.	Semi-circle		$\frac{4r}{3\pi}$	$\frac{d}{2}$	$\frac{1}{2} \times \frac{\pi d^2}{4}$
8	Quadrant		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{1}{4} \pi r^2$.

		$\bar{x}$	$\bar{y}$	Area
9. Quadrant		$r - \frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{1}{4} \times \pi r^2$
10. Trapezium		$\frac{b}{2}$	$\left(\frac{b+2a}{b+a}\right)\frac{h}{3}$	$\frac{1}{2}(a+b)h$

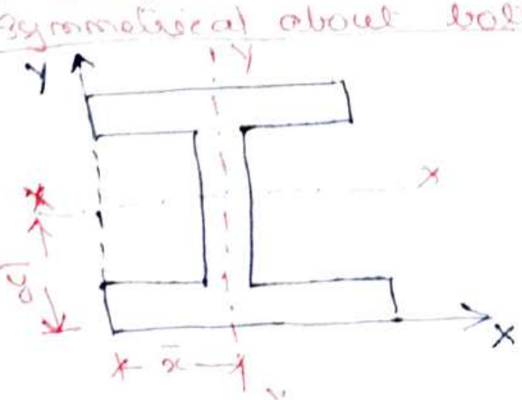
Centroid of composite plane figures:

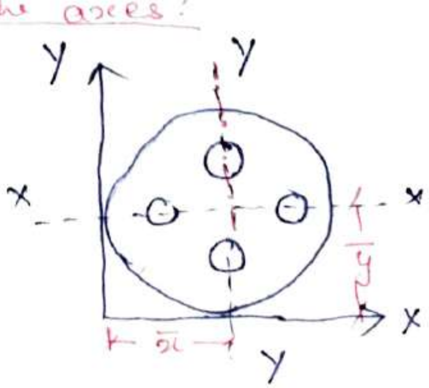
If a plane figure is a combination of two or more simple plane figures, the algebric sum of moments of the individual area about any axis of reference will be equal to the moment of the whole area about the same axis. Hence, the centroid of the composite plane figures are determined by the method of moments.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + \cdots + a_nx_n}{a_1 + a_2 + \cdots + a_n} \quad \text{lly} \quad \bar{y} = \frac{a_1y_1 + a_2y_2 + \cdots + a_ny_n}{a_1 + a_2 + \cdots + a_n}$$

Axis of symmetry: If a composite plane figure has an axis of symmetry (i.e an axis about which similar configuration is seen on either side), centroid lies on it. This concept reduce the work of locating the centroid.

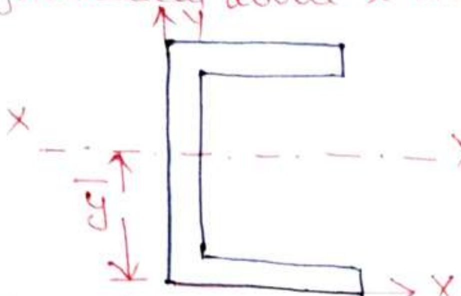
i) Symmetrical about both the axes:

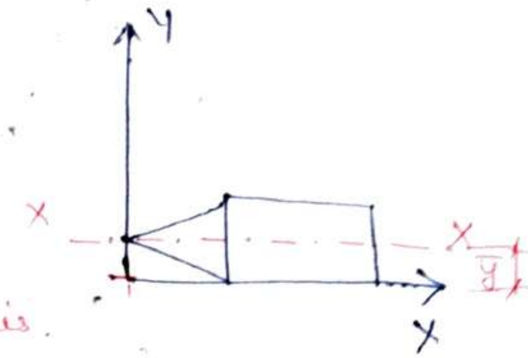




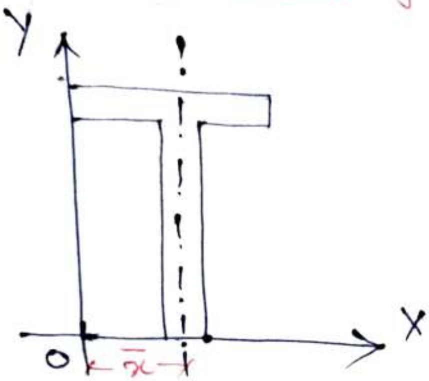
Here no calculation is required to locate the centroid.

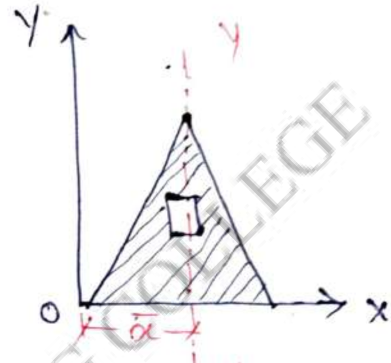
ii) Symmetrical about x-axis.



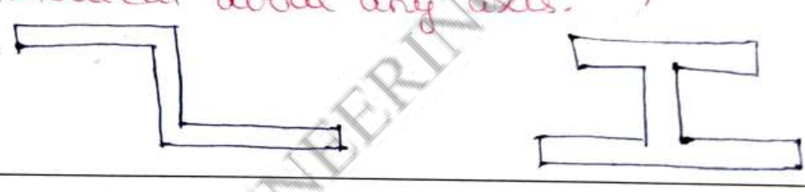


iii) symmetrical about y-axis.





iv) Not symmetrical about any axis.



Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston Jr., David F Mazurek, Philip J Cornwell, Sanjeev Sanghi,	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press

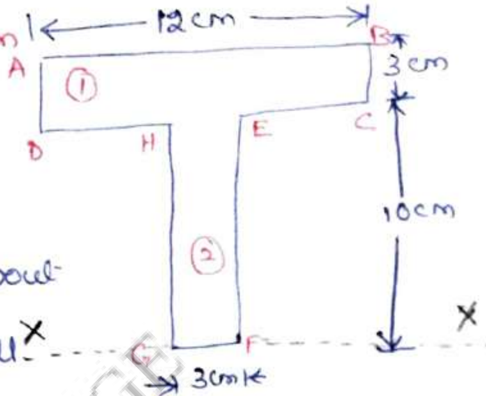
Any other suggested Materials

Class notes and Handouts

Centroid of T-Section, I-Section, Angle-Section, Hollow-Section etc.

The centroid of structural sections like T-Sec, I-Sec. etc. are obtained by splitting them into rectangular components.

1. Find the centre of gravity of the T-Section shown in figure.



1) Split into rectangular portions.

2) The given T-section is symmetric about YY axis. Hence C.G. of the Section will lie on this axis.

3) The lowest line of figure is GF. Hence fix this as the axis of reference.

Let $\bar{y}$ = = distance of C.G. of T-sec. from the bottom line GF. (axis of reference).

a_1 = area of ABCD = $12 \times 3 = 36\ cm^2$.

y_1 = Distance of C.G. of a_1 from GF.

$= 10 + \frac{3}{2} = 11.5\ cm.$

$a_2 = 10 \times 3 = 30\ cm^2$

$y_2 = \frac{10}{2} = 5\ cm.$

$\boxed{A\bar{y} = a_1y_1 + a_2y_2}$

we know that,

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{A} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}.$$

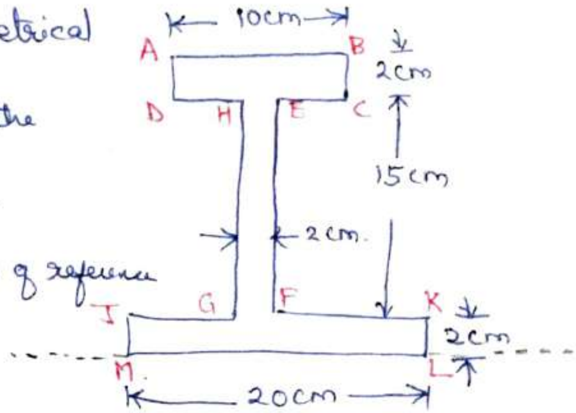
$$= \frac{(36 \times 11.5) + (30 \times 5)}{36 + 30} = \frac{414 + 150}{66}$$

$$\boxed{\bar{y} = 8.545\ cm.}$$

2. Find the centre of gravity of the I-section shown in fig.

The given I-section is symmetrical about Y-Y axis. Hence C.G. of the section will lie on this axis.

lowest line of fig. is ML – axis of reference



$$\boxed{\bar{y}A = a_1y_1 + a_2y_2 + a_3y_3}$$

$a_1 = 10\times2 = 20\,cm^2$; $a_2 = 15\times2 = 30\,cm^2$; $a_3 = 20\times2 = 40\,cm^2$.

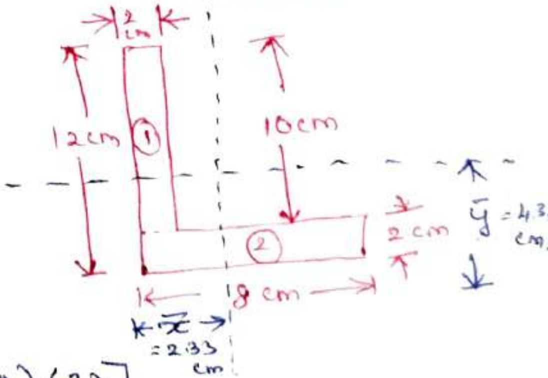
$y_1 = 2+15+\frac{2}{2} = 18cm$; $y_2 = 2+\frac{15}{2} = 9.5\,cm$. ; $y_3 = 1cm$.

$$\bar{y} = \frac{(20\times18)+(30\times9.5)+(40\times1)}{20+30+40}$$

$$= \frac{360+285+40}{90} = \frac{685}{90} = 7.611\,cm.$$

3. Find the centre of gravity of L-section shown.

The given L-section is not symmetric about any axis.



$$\boxed{A\bar{y} = a_1y_1 + a_2y_2}$$

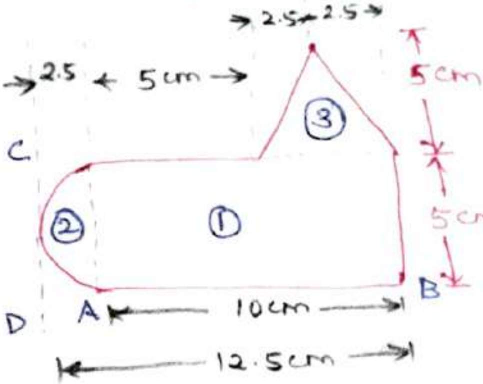
$$\bar{y} = \frac{a_1y_1 + a_2y_2}{A} = \cancel{(10\times2)}$$

$$= \frac{\left[(10\times2)\left(2+\frac{10}{2}\right)\right] + \left[(8\times2)\left(\frac{2}{2}\right)\right]}{(10\times2)+(8\times2)}$$

$$= \frac{(20\times7)+(16\times1)}{20+16} = \frac{140+16}{36} = \frac{156}{36} = 4.33\,cm.$$

$$\bar{x} = \frac{a_1x_1+a_2x_2}{A} = \frac{(20\times1)+(16\times4)}{20+16.} = 2.33\,cm.$$

4. Using the analytical method, determine the C.G. of the plane uniform lamina shown in figure. (university ques.)



Area 1: $a_1 = 10 \times 5\ cm^2 = 50\ cm^2$

$y_1 = \frac{5}{2} = 2.5\ cm$

Area 2: $a_2 = \frac{\pi r^2}{2} = \frac{\pi (2.5)^2}{2}$

$= 9.82\ cm^2$

$y_2 = \frac{5}{2} = 2.5\ cm.$

Area 3:

$a_3 = \frac{1}{2} \times 5 \times 5 = 12.5\ cm^2$

$y_3 = 5 + \frac{5}{3} = 6.67\ cm.$

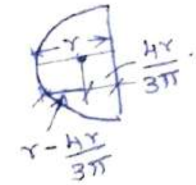
Now, $\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(50 \times 2.5) + (9.82 \times 2.5) + (12.5 \times 6.67)}{50 + 9.82 + 12.5}$

$= \frac{232.9}{72.32} = 3.22\ cm.$

Let $\bar{x}$ be the distance of C.G. of lamina from left line.

$a_1 = 50\ cm^2$; $a_2 = 9.82\ cm^2$; $a_3 = 12.5\ cm^2$;

$x_1 = 2.5 + \frac{10}{2} = 7.5\ cm.$

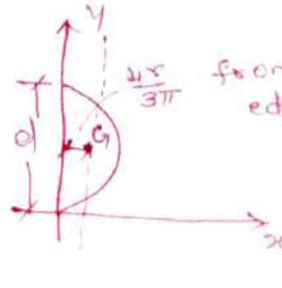


$x_2 = 2.5 - \frac{4r}{3\pi} = 2.5 - \frac{4.25}{3\pi} = 1.44\ cm.$

$x_3 = 2.5 + 5 + 2.5 = 10\ cm.$

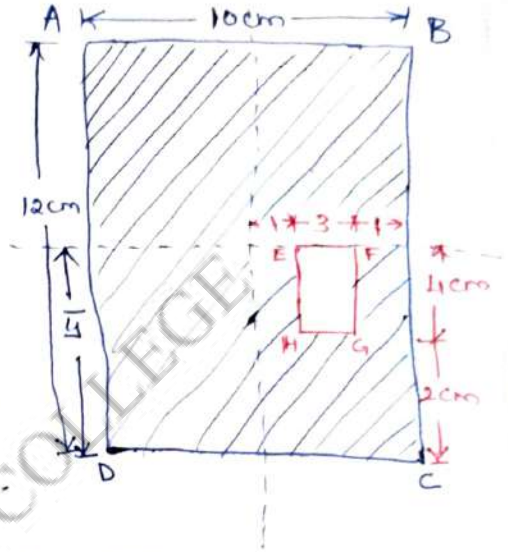
$\bar{x} = \frac{(50 \times 7.5) + (9.82 \times 1.44) + (12.5 \times 10)}{50 + 9.82 + 12.5} = \frac{514.14}{72.32} = 7.11\ cm.$

Hence C.G. of uniform lamina is at a distance of 3.22 cm from bottom line AB and 7.11 cm from left line CD.



5. From the rectangular lamina ABCD 10cm × 12 cm, a rectangular hole of 3cm × 4cm is cut as shown in fig. Find the C.G. of the remainder lamina.

Let $\bar{y}$ is the distance between the C.G. of the section with a cut hole from bottom line DC.



Reference line: Bottom of rectangle DC.

Area 1: a_1 = area of rectangle ABCD.

$= 10 \times 12 = 120\ cm^2$

y_1 = distance of C.G. of the rectangle ABCD from bottom line DC, $= \frac{12}{2} = 6cm$.

Area 2:

a_2 = area of cut-out hole i.e. rectangle EFGH.

$= 4 \times 3 = 12\ cm^2$

y_2 = Distance of C.G. of cut-out hole from bottom line DC

$= 2 + \frac{4}{2} = 2+2 = 4cm$.

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{A} \quad \text{where } A = a_1 - a_2.$$

$$= \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(120 \times 6) - (12 \times 4)}{120 - 12} = \frac{720 - 48}{108} = 6.22\ cm$$

To find $\bar{x}$, let $\bar{x}$ = distance between C.G. of the section with a cut hole from left line AD.

$x_1 = \frac{10}{2} = 5\ cm \quad ; x_2 = 5 + 1 + \frac{3}{2} = 7.5\ cm$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(120 \times 5) - (12 \times 7.5)}{120 - 12} = \frac{600 - 90}{108}$$

$$= \frac{510}{108} = 4.72\ cm.$$

Hence C.G. of the section with a cut hole will be at a distance of 6.22 cm from bottom line DC and 4.72 cm from the line AD.

Lecture No. 22 **UNIT III – DISTRIBUTED FORCES**

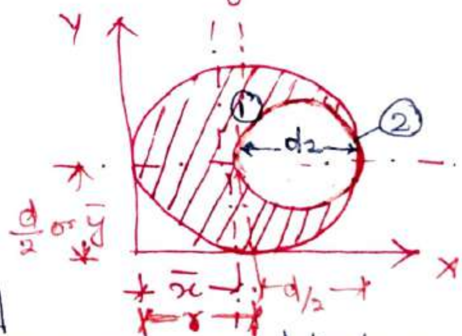
Topic(s) to be covered	Locating centroid.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	Locate centroid for composite sections.	L3

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and solve.

Lecture Notes

(6) A circular hole is punched out of a circular lamina as shown. The diameter of the circular hole which is punched out is equal to the radius of the circular plate. Find the centroid of the remaining lamina.



Portion 1: circular plate:

$a_1 = \pi r_1^2$ or $\frac{\pi d_1^2}{4}$.

$x_1 = r_1$ or $\frac{d_1}{2}$.

Portion 2: circular hole. $\boxed{d_2 = r_1}$

[The dia of punched hole = radius of the circular plate]

$a_2 = \pi \frac{d_2^2}{4} = \frac{\pi r_1^2}{4}$; $x_2 = r + \frac{r}{2}$ $\boxed{x_2 = \frac{3r}{2}}$

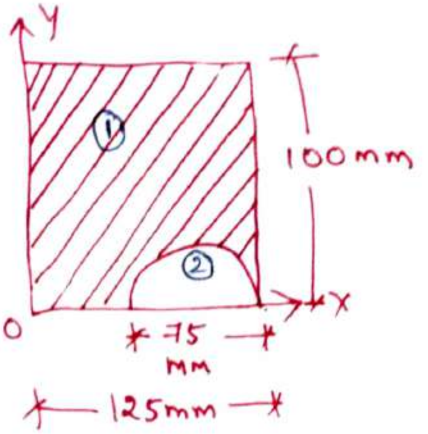
$\boxed{\bar{y} = \frac{d_1}{2}}$ $\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(\pi r^2 \times r) - \left(\frac{\pi r^2}{4} \times \frac{3r}{2}\right)}{\pi r^2 - \frac{\pi r^2}{4}}$

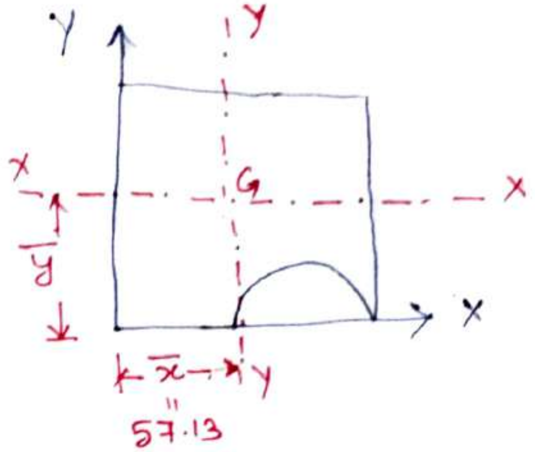
$\bar{x} = \frac{\pi r^2 \left(r - \frac{3r}{8}\right)}{\pi r^2 (1 - 1/4)} = \frac{5r/8}{3/4} = \frac{5r}{8} \times \frac{4}{3}$

$= \frac{5r}{6} = 0.833r.$

$\boxed{\bar{x} = 0.833r}$ $y = \frac{d}{2} = \frac{2r}{2} \Rightarrow \boxed{y = r}$

(7) Locate the centroid of the lamina shown.





The section shown above is having a cut portion of a semicircle of 75 mm diameter.

Portion (1) - Rectangle:

$$a_1 = 125 \times 100 = 12500\ mm^2$$

$$x_1 = \frac{125}{2} = 62.5\ mm \quad ; \quad y_1 = \frac{100}{2} = 50\ mm.$$

Portion (2) - Semi-circle:

$$a_2 = \frac{1}{2} \times \frac{\pi d^2}{4} = \frac{1}{2} \times \frac{\pi \times 75^2}{4} = 2208.93\ mm^2.$$

$$x_2 = 125 - \frac{75}{2} = 87.5\ mm. \quad ; \quad y_2 = \frac{4r}{3\pi} = \frac{4 \times 37.5}{3\pi}$$

$$\boxed{y_2 = 15.91mm.}$$

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 + a_2}.$$

$$= \frac{(12500 \times 62.5) - (2208.93 \times 87.5)}{12500 - 2208.93} \Rightarrow \boxed{\bar{x} = 57.13\ mm}$$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(12500 \times 50) - (2208.93 \times 15.91)}{(12500 - 2208.93)}$$

$$\boxed{\bar{y} = 57.31\ mm}$$

Lecture No. 23.

UNIT III – DISTRIBUTED FORCES

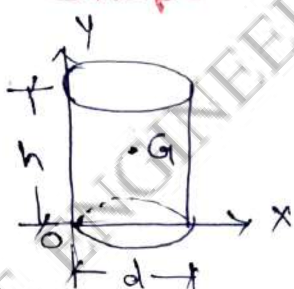
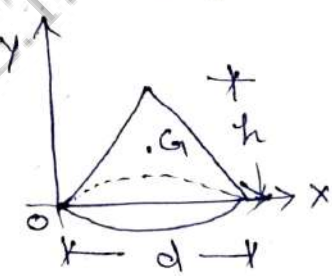
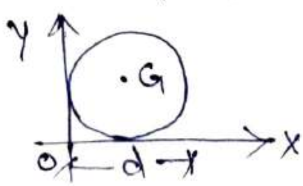
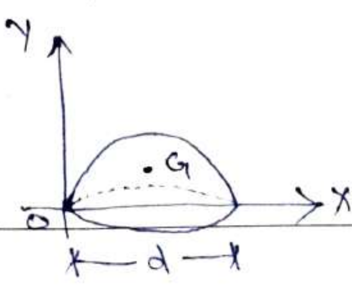
Topic(s) to be covered	centre of gravity of solid figures.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	locate the centre of gravity of solid figures.	L3.

Teaching Learning Material	Student Activity
chalk and Talk.	Learn and solve.

Lecture Notes

Centre of gravity of solid figures:

S.No.	Name	Shape	$\bar{x}$	$\bar{y}$	Volume
1.	cylinder		$\frac{d}{2}$	$\frac{h}{2}$	$\pi r^2 h$.
2.	cone		$\frac{d}{2}$	$\frac{h}{4}$	$\frac{1}{3}\pi r^2 h$.
3.	sphere		$\frac{d}{2}$	$\frac{d}{2}$	$\frac{4}{3}\pi r^3$.
4.	Hemi-sphere		$\frac{d}{2}$	$\frac{3r}{8}$	$\frac{2}{3}\pi r^3$.

Centre of gravity of composite solid figures:

The centre of gravity of solid figures are determined by integration and method of moments.

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + \dots + w_n} \quad \text{---- (i)}$$

and $$\bar{y} = \frac{w_1y_1 + w_2y_2 + \dots + w_ny_n}{w_1 + w_2 + \dots + w_n} \quad \text{---- (ii)}$$

Case (i): If the composite solid is made of same material (i.e. density values are same), equations (i) and (ii) are modified as follows.

$$\bar{x} = \frac{V_1x_1 + V_2x_2 + \dots + V_nx_n}{V_1 + V_2 + \dots + V_n} \text{ and } \bar{y} = \frac{V_1y_1 + V_2y_2 + \dots + V_ny_n}{V_1 + V_2 + \dots + V_n}$$

where, V_1 and V_2 are the volumes of components 1 and 2 respectively.

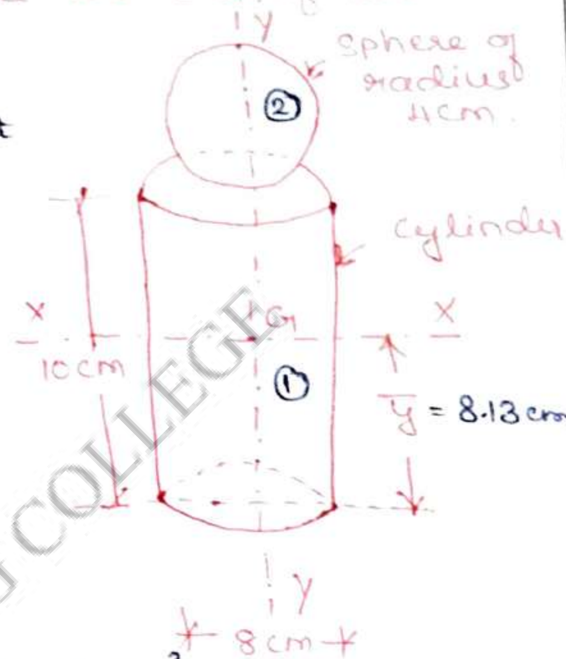
Case (ii): If the composite solid is made of different materials (i.e, density values are different), equations (i) and (ii) are modified as follows,

$$\bar{x} = \frac{(\rho_1V_1)x_1 + (\rho_2V_2)x_2 + \dots + (\rho_nV_n)x_n}{(\rho_1V_1) + (\rho_2V_2) + \dots + (\rho_nV_n)}$$

and $$\bar{y} = \frac{(\rho_1V_1)y_1 + (\rho_2V_2)y_2 + \dots + (\rho_nV_n)y_n}{(\rho_1V_1) + (\rho_2V_2) + \dots + (\rho_nV_n)}$$

where ρ_1 and ρ_2 are the density of components 1 and 2 respectively.

A cylinder of height 10cm and radius of base 4cm is placed under sphere of radius 4cm such that they have a common vertical axis. If both of them are made of the same material, locate the C.G. of the combined unit.



The composite solid is ~~vertical~~ symmetrical about vertical axis;

Hence $\bar{x} = \frac{8}{2} = 4\,cm.$

$$\boxed{\bar{x} = 4\,cm}$$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$

Solid (1) : cylinder.

$$V_1 = \pi r^2 h = \pi \times 4^2 \times 10 = 502.65\ cm^2.$$

$$y_1 = \frac{10}{2} = 5\,cm.$$

Solid (2): sphere

$$V_2 = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 4^3 = 268\ cm^3.$$

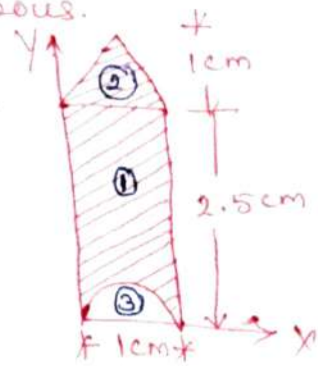
$$y_2 = 10 + 4 = 14\,cm.$$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = \frac{(502.65 \times 5) + (268 \times 14)}{(502.65 + 268)}$$

$$\boxed{\bar{y} = 8.13\,cm}$$

$$\boxed{\bar{y} = 7.43\,cm}$$

→ Locate the CG of a bullet, 1cm diameter with a cone on the front and a hemisphere cut from the back as shown. Assume the material to be homogeneous.



Due to symmetry; $\bar{x} = \frac{1}{2} \Rightarrow \boxed{\bar{x} = 0.5\,cm}$

Portion 1: cylinder

$V_1 = \pi r^2 h = \pi (0.5)^2 \times 2.5 = 1.963\ cm^3$

$y_1 = \frac{2.5}{2} = 1.25\ cm.$

Portion 2: cone

$V_2 = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (0.5)^2 \times 1 = 0.261\ cm^3.$

$y_2 = 2.5 + \left(\frac{1}{4}\right) = 2.75\ cm.$

Portion 3: Hemisphere: $V_3 = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(0.5)^3 = 0.261\ cm^3.$

$y_3 = \frac{3r}{8} = \frac{3 \times 0.5}{8} = 0.1875\ cm.$

$\bar{y} = \frac{V_1 y_1 + V_2 y_2 - V_3 y_3}{V_1 + V_2 - V_3} = \frac{(1.963 \times 1.25) + (0.261 \times 2.75) - (0.261 \times 0.1875)}{(1.963 + 0.261 - 0.261)}$

$\boxed{\bar{y} = 1.59\ cm}$

Suggested Questions / Assignments / Home works / any other

H.W: ① P-427, Qn 1. ② P-430, Qn. 4.

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston Jr., David F Mazurek, Philip J Cornwell, Sanjeev Sanghi,	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press

Any other suggested Materials

Class notes and Handouts

Lecture No. 24.

UNIT III – DISTRIBUTED FORCES

Topic(s) to be covered	Pappus and Guldinus theorem.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	Understand and apply the concept of Pappus and Guldinus theorem.	L3.

Teaching Learning Material	Student Activity
Chalk and Talk.	Learn and solve.

Lecture Notes

Pappus and Guldinus theorem:

Theorem 1: It states that "the area of a surface of revolution is the product of the length of the generating curve and the distance travelled by the centroid of the curve, while the surface is generated."

$$\left.\begin{array}{c}\text{Surface area}\\\text{generated}\end{array}\right\} = \left\{\begin{array}{c}\text{Distance travelled}\\\text{by the centroid}\\\text{of generating arc}\end{array}\right\} \times \left\{\begin{array}{c}\text{length}\\\text{of}\\\text{arc}\end{array}\right\}$$

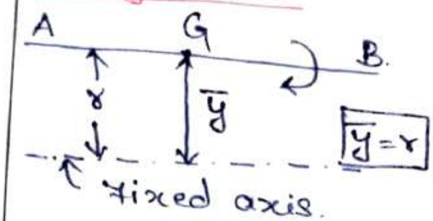
Theorem 2: It states that "the volume of a body of revolution is obtained from the product of the generating area and the distance travelled by the centroid of the area, while the body is being generated".

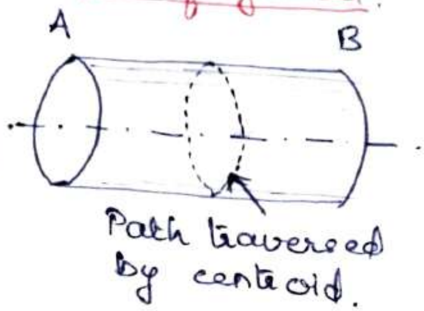
$$\left.\begin{array}{c}\text{Volume}\\\text{generated}\end{array}\right\} = \left\{\begin{array}{c}\text{Distance travelled by}\\\text{centroid of generating}\\\text{area}\end{array}\right\} \times \left\{\begin{array}{c}\text{Generating}\\\text{area}\end{array}\right\}$$

Pappus-Guldinus theorem 1

Generating curve	Surface Generated	Distance travelled by the centroid	Length of arc

Straight Line — **Surface of cylinder**



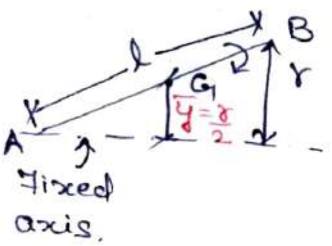


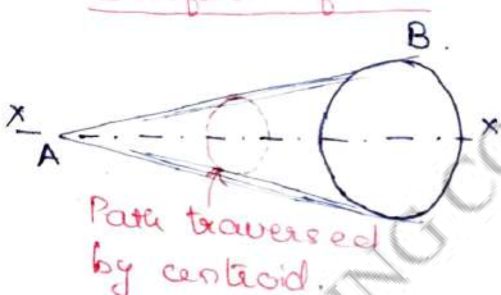
$2\pi\bar{y}$
$= 2\pi r$

l

$\therefore$ Surface area generated $= 2\pi r l$.

Straight Line — **Surface of cone**



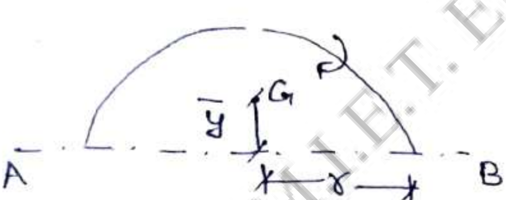


Distance travelled
$= 2\pi\bar{y}$
$= 2\pi\left(\frac{r}{2}\right)$
$= \pi r$.

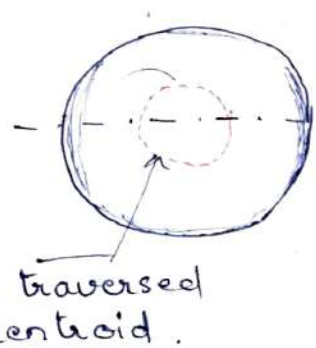
l.

$\therefore$ Surface area generated $= \pi r l$.

Semi-circular arc — **Surface of sphere**



$$\boxed{\bar{y} = \frac{2r}{\pi}}$$



Distance travelled
$= 2\pi\bar{y}$
$= 2\pi\left(\frac{2r}{\pi}\right)$
$= 4r$.

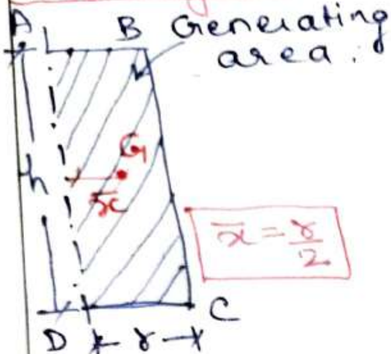
$\frac{2\pi r}{2}$
$= \pi r$

$\therefore$ Surface area generated
$= 4r \times \pi r$
$= 4\pi r^2$

Pappus - Guldinus Theorem 2

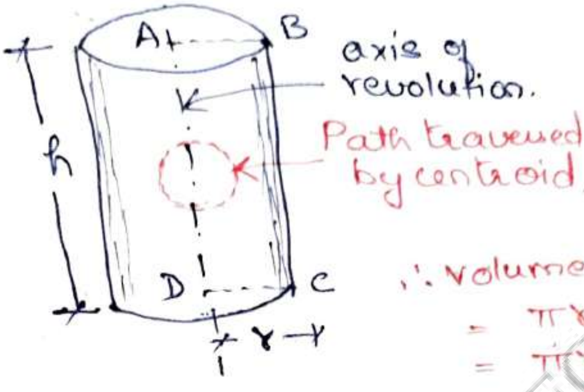
Generating area	Volume Generated	Distance travelled by centroid	Generating area

Rectangle



$\boxed{\bar{x} = \frac{r}{2}}$

Cylinder



$2\pi\bar{x}$

$= 2\pi\left(\frac{r}{2}\right)$ $\quad rh$

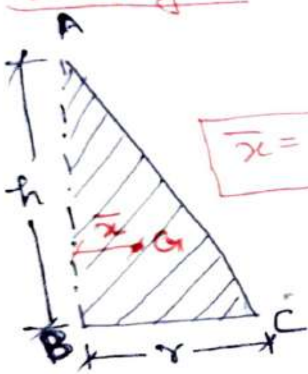
$= \pi r$.

$\therefore$ Volume Generated

$= \pi r \times rh$

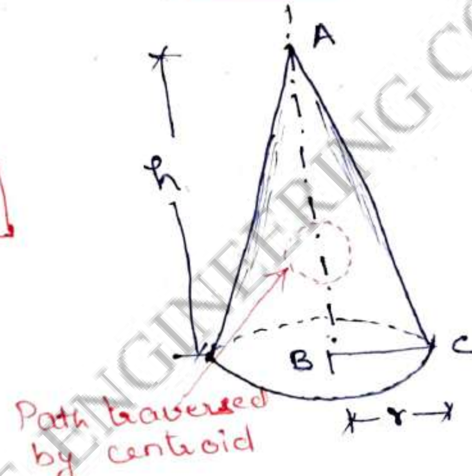
$= \pi r^2 h$.

Right-angled triangle



$\boxed{\bar{x} = \frac{r}{3}}$

cone



$2\pi\bar{x}$ $\quad \frac{1}{2}rh$

$= 2\pi\left(\frac{r}{3}\right)$

$= \frac{2\pi r}{3}$

$\therefore$ Volume Generated

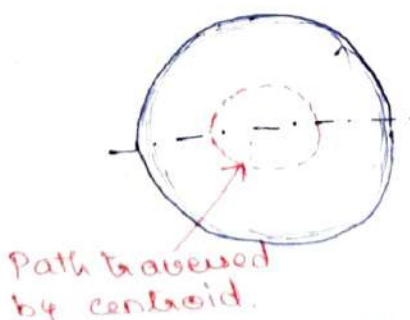
$= \frac{2}{3}\pi r \times \frac{1}{2} rh$

$= \frac{1}{3}\pi r^2 h$.

Semi-circle

$\boxed{\bar{x} = \frac{4r}{3\pi}}$

Sphere



$2\pi\bar{x}$ $\quad \frac{\pi r^2}{2}$

$= 2\pi\left(\frac{4r}{3\pi}\right)$

$= \frac{8r}{3}$

$\therefore$ Volume Generated

$= \frac{8r}{3} \times \frac{\pi r^2}{2}$

$= \frac{4}{3}\pi r^3$.

Determine the surface area of a cone by Pappus-Guldinus theorem.

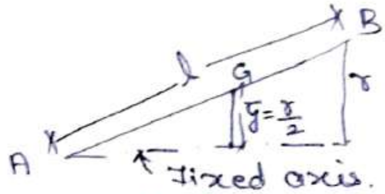


fig (a)

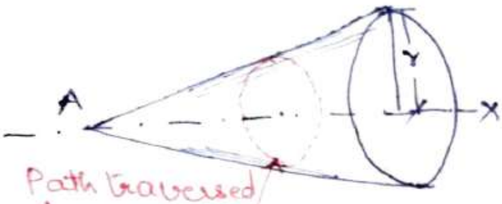


fig (b).

Let an inclined line AB of length 'l' is rotated about a fixed axis AX (known as axis of revolution with a distance of end B from AX axis as 'r'. The surface area generated is a cone as shown in fig (b).

Centroid of the line is at the midpoint of AB.

Hence, height of centroid from fixed axis $\bar{y} = \frac{r}{2}$

$\therefore$ Distance travelled (i.e. circumference) by centroid of the line. $= 2\pi\bar{y} = 2\pi\left(\frac{r}{2}\right) = \pi r$

Applying Pappus-Guldinus theorem 1:

$$\left.\begin{array}{l}\text{Surface area}\\ \text{generated}\end{array}\right\} = \left\{\begin{array}{l}\text{Distance travelled by the}\\ \text{centroid of generating arc}\end{array}\right\} \times \left\{\begin{array}{c}\text{length of}\\ \text{arc}\end{array}\right\}$$

$$= \pi r \times l$$

$$= \pi r l.$$

Suggested Questions / Assignments / Home works / any other
Determine the surface area and volume of other solids in the table.

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press
Any other suggested Materials			
Class notes and Handouts			

Lecture No. **UNIT III – DISTRIBUTED FORCES**

Topic(s) to be covered	Moment of Inertia

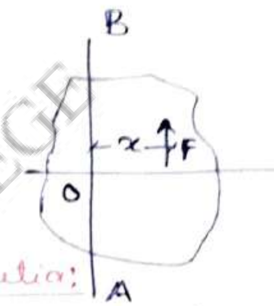
	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

First moment of force: The moment of force F about O, on axis AB,

$M_O = F \times x.$



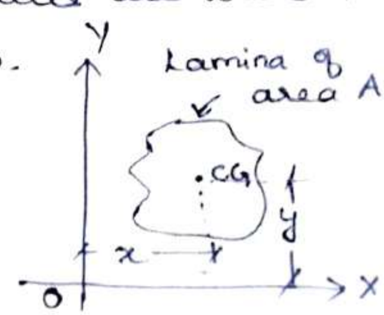
Second moment of force or Moment of Inertia:

If first moment of force is again multiplied by x, then it is called as second moment of force or moment of inertia.

MOI = First moment × distance

$= (F \times x) \times x = Fx^2.$

Sometimes, plane and solid figures are taken into consideration instead of the force.



1st moment of area:

Moment of area about the axis OY

= area × ⊥r. distance of C.G of area from axis OY

$= A.x.$

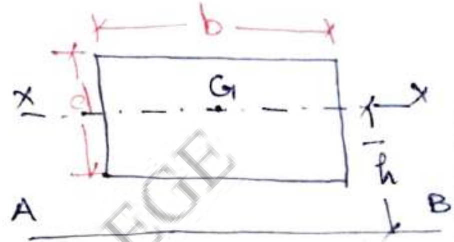
The first moment of area is used to determine the centroid of the area.

2nd moment of area: If the moment of area is again multiplied by the ⊥r. distance, then the quantity Ax^2 is known as moment of the moment of area or second moment of area or Moment of inertia.

Parallel axis theorem:

It states that "the moment of inertia of a lamina about any axis in the plane of lamina is equal to the sum of MOI about a parallel centroidal axes in the plane of lamina and the product of the area of the lamina and square of the distance between the two axes

$$\boxed{I_{AB} = I_G + A\bar{h}^2}$$

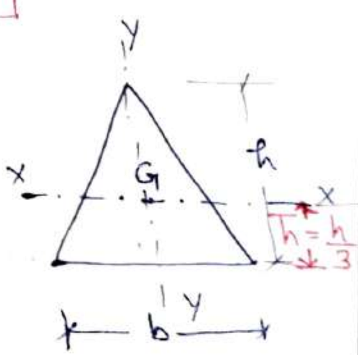


For a rectangular section:

$$I_{xx} = \frac{bd^3}{12}$$

$$I_{AB} = \frac{bd^3}{12} + \left[(bd)\left(\frac{d}{2}\right)^2\right]$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4} \Rightarrow \boxed{\underset{@\,base}{I_{AB}} = \frac{bd^3}{3}}$$



For a triangular section:

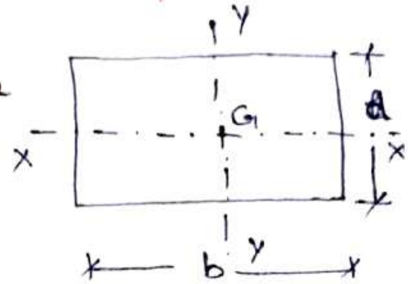
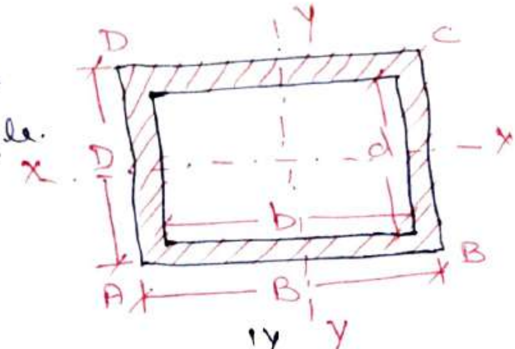
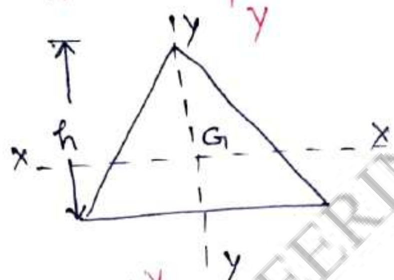
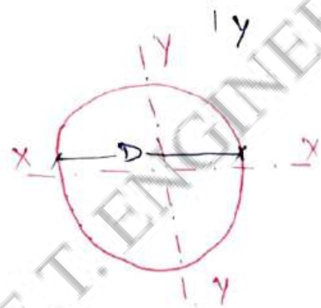
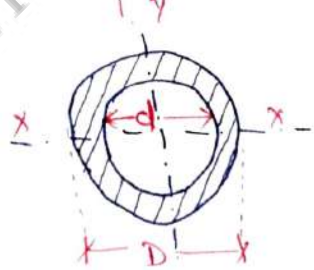
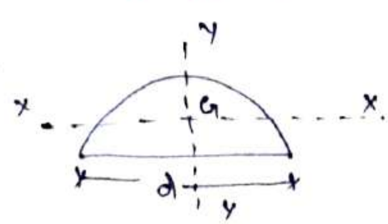
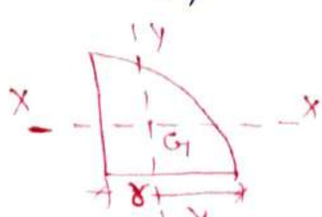
$$I_{xx} = \frac{bh^3}{36}.$$

$$I_{AB} = I_G + A\bar{h}^2$$

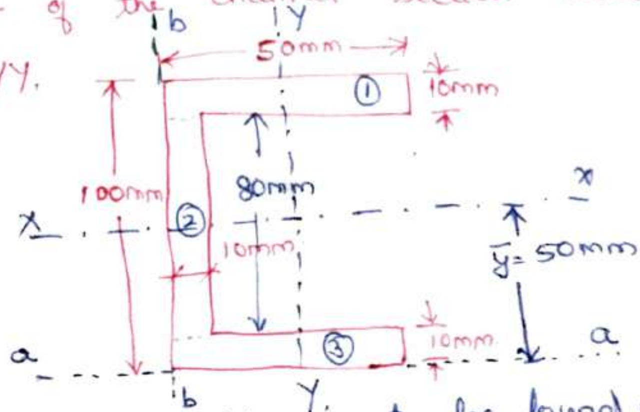
$$= \frac{bh^3}{36} + \left[\left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2\right]$$

$$= \frac{bh^3}{36} + \frac{bh^3}{18} \Rightarrow I_{AB} = \frac{bh^3 + 2bh^3}{36} \Rightarrow \boxed{\underset{@\,base}{I_{AB}} = \frac{bh^3}{12}}$$

Moment of Inertia of common simple plane figures.

Sl.No.	Name	Shape	I_{xx}	I_{yy}	I_{AB}
1.	Rectangle		$\frac{bd^3}{12}$	$\frac{db^3}{12}$	$\frac{db^3}{3}$.
2.	Hollow Rectangle.		$\frac{BD^3}{12} - \frac{bd^3}{12}$ $\frac{1}{12}[BD^3 - bd^3]$	$\frac{1}{12}[DB^3 - db^3]$	
3.	Triangle.		$\frac{bh^3}{36}$	$\frac{hb^3}{48}$	$I_{AB} = \frac{bh^3}{12}$
6.	Circle.		$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$	
7.	Hollow circle.		$\frac{\pi}{64}(D^4 - d^4)$	$\frac{\pi}{64}(D^4 - d^4)$.	
8.	Semi-circle.		$0.0068d^4$	$\frac{\pi d^4}{128}$	
9.	Quadrant.		$0.055r^4$	$0.055r^4$.	

Find the MOI of the channel section about its centroidal axes XX and YY.



First, the C.G. of this section is to be found out.

$$\bar{x} = \frac{(a_1x_1) + (a_2x_2) + (a_3x_3)}{(a_1 + a_2 + a_3)}.$$

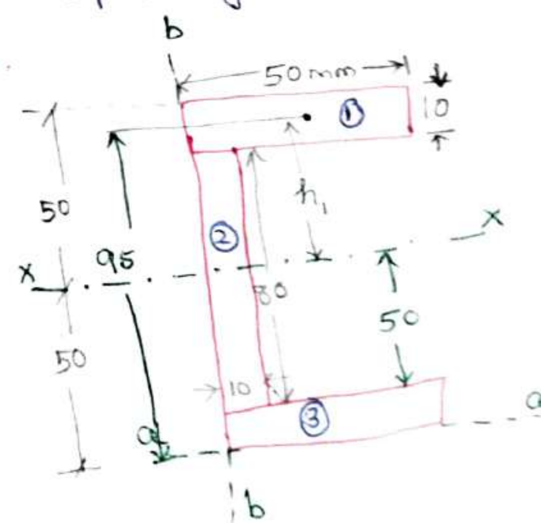
$$= \frac{(50\times10\times25) + (10\times80\times5) + (50\times10\times25)}{(500 + 800 + 500)}.$$

$$= \frac{29000}{1800} \Rightarrow \boxed{\bar{x} = 16.11\,mm}$$ $\boxed{A = 1800\,mm^2}$

Due to symmetry about XX axis, $\boxed{\bar{y} = 50\,mm.}$

consider two reference axes 'aa' and 'bb' at the bottom and left-hand edges of the section respectively.

To find MOI about axis XX (I_{xx}).



MOI of 1st rectangle about axis XX

$$MI_1 = I_{self} + Ah^2 = \frac{b_1d_1^3}{12} + A_1h_1^2$$

$$= \frac{50\times10^3}{12} + (50\times10)(95-50)^2$$

$$= 4166.67 + 1012500.$$

$$= 1016666.67\,mm^4.$$

MOI of the 2nd rectangle about axis XX,

$$MI_2 = I_{self} + Ah^2 = \frac{b_2d_2^3}{12} + A_2h_2^2$$

$$= \frac{10\times80^3}{12} + (10\times80)(50-50)^2$$

$$= 426666.67\,mm^4.$$

$|||^{ly}$ the MOI of 3rd rectangle about axis xx,

$$MI_3 = I_{self} + Ah^2 = \frac{b_3 d_3^3}{12} + A_3 h_3^2.$$

$$MI_3 = \frac{50 \times 10^3}{12} + (50 \times 10)(50-5)^2$$

$$= 4166.67 + 1012500$$

$$= 1016666.67 \text{ mm}^4.$$

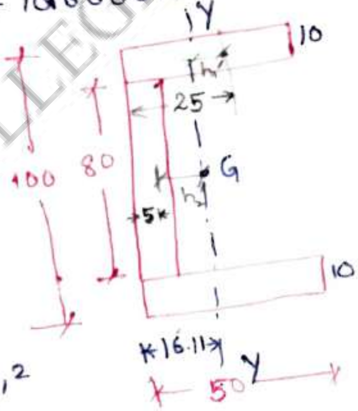
MOI of the entire section about xx, is

$$I_{xx} = MI_1 + MI_2 + MI_3.$$

$$= 1016666.67 + 426666.67 + 1016666.67.$$

$$= 2460000.01 \text{ mm}^4.$$

$$\boxed{I_{xx} = 2.46 \times 10^6 \text{ mm}^4}$$



To find MOI about axis YY: (I_{yy})

MOI of 1st rectangle about axis YY,

$$MI_1 = I_{self} + Ah^2 = \frac{d_1 b_1^3}{12} + A_1 h_1'^2.$$

$$= \frac{10 \times 50^3}{12} + (10 \times 50)(25 - 16.11)^2$$

$$= 104166.67 + 39516.05$$

$$= 143682.72 \text{ mm}^4.$$

MOI of 2nd rectangle about axis YY,

$$MI_2 = I_{self} + Ah^2 = \frac{d_2 b_2^3}{12} + A_2 h_2'^2$$

$$= \frac{80 \times 10^3}{12} + (80 \times 10)(16.11-5)^2$$

$$= 6666.67 + 98745.68.$$

$$= 105412.35 \text{ mm}^4.$$

$|||^{ly}$ $MI_3 = MI_1 = 143682.72 \text{ mm}^4.$

MOI of entire section about YY axis,

$$I_{yy} = MI_1 + MI_2 + MI_3.$$

$$= 143682.72 + 105412.35 + 143682.72$$

$$= 392777.79 \text{ mm}^4. \quad \boxed{I_{yy} = 0.39 \times 10^6 \text{ mm}^4}$$

Polar moment of inertia $I_{zz} = I_{xx} + I_{yy}$.

$$= (2.46 + 0.39)\,10^6$$

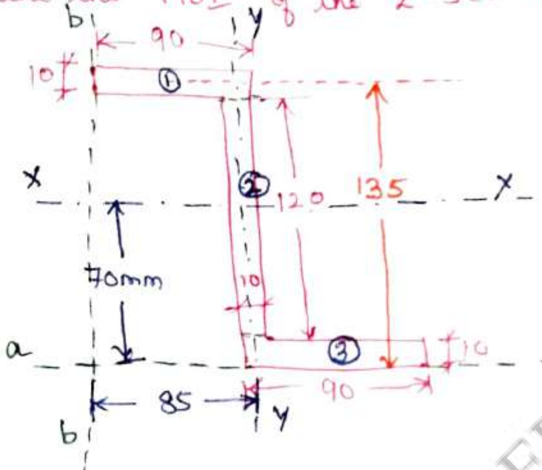
$$= 2.85 \times 10^6 \ mm^4.$$

Radius of gyration:

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{2.46 \times 10^6}{1800}} = 36.97 \ mm.$$

$$k_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{0.39 \times 10^6}{1800}} = 14.72 \ mm.$$

Determine the MOI of the Z-section about its centroidal axes.



consider two reference axes 'aa' and 'bb' at the bottom and left hand edges of section respectively.

To find MOI of the section about XX axis

MOI MI_1 of 1st rectangle about axis XX is,

$$MI_1 = I_{self} + Ah^2$$

$$= \frac{b_1 d_1^3}{12} + A_1 h_1^2$$

$$= \frac{90 \times 10^3}{12} + (90 \times 10)(135-70)^2.$$

$$= 7500 + 3802500.$$

$$= 3810000 \ mm^4.$$

MOI MI_2 of the second rectangle about axis XX is,

$$MI_2 = I_{self} + Ah^2$$

$$= \frac{b_2 d_2^3}{12} + A_2 h_2^2$$

$$= \frac{10 \times 120^3}{12} + (10 \times 120)(\cancel{70}-70)^2.$$

$$= 1440000 \ mm^4.$$

MOI MI_3 of 3^{rd} rectangle about xx,

$$MI_3 = I_{self} + Ah^2$$

$$= \frac{b_3 d_3^3}{12} + A_3 h_3^2$$

$$= \frac{90 \times 10^3}{12} + (90 \times 10)(70-5)^2 .$$

$$= \cancel{1440000} \quad 7500 + 3802500$$

$$= 3810000 \text{ mm}^4 .$$

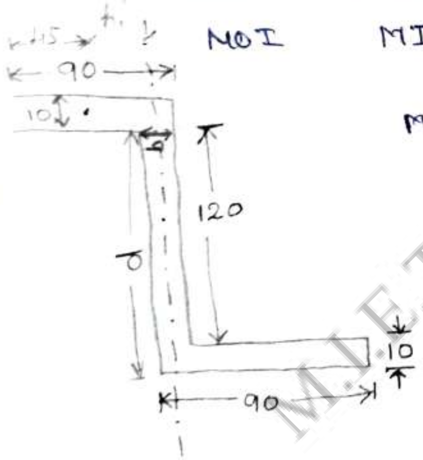
MOI I_{xx} of the entire section about xx-axis is

$$I_{xx} = MI_1 + MI_2 + MI_3$$

$$= 3810000 + 1440000 + 3810000$$

$$= 9060000 \text{ mm}^4 .$$

To find the MOI about yy axis:



MOI MI_1 of 1^{st} rectangle about axis yy is

$$MI_1 = I_{self} + Ah^2 .$$

$$= \frac{d_1 b_1^3}{12} + A_1 h_1'^2$$

$$= \frac{10 \times 90^3}{12} + (10 \times 90)(85-45)^2$$

$$= 607500 + 1440000 .$$

$$= 2047500 \text{ mm}^4 . = MI_3$$

MOI MI_2 of the 2^{nd} rectangle about yy axis is,

$$MI_2 = I_{self} + Ah^2$$

$$= \frac{d_2 b_2^3}{12} + A_2 h_2'^2$$

$$= \frac{120 \times 10^3}{12} + (120 \times 10)(85-85)^2$$

$$= 10000 \text{ mm}^4 .$$

MOI MI_3 of 3^{rd} rectangle about yy axis,

$$MI_3 = I_{self} + Ah^2$$

$$= \frac{10 \times 90^3}{12} + (10 \times 90)(125-85)^2$$

$$= 607500 + 1440000$$

$$= 2047500 \text{ mm}^4 .$$

Finally MOI I_{yy} of the entire section about yy axis is $I_{yy} = MI_1 + MI_2 + MI_3$

$$= 2047500 + 10000 + 2047500 = 4105000 \text{ mm}^4 = 4.1 \times 10^6 \text{ mm}^4$$

Lecture No. **UNIT III – DISTRIBUTED FORCES**

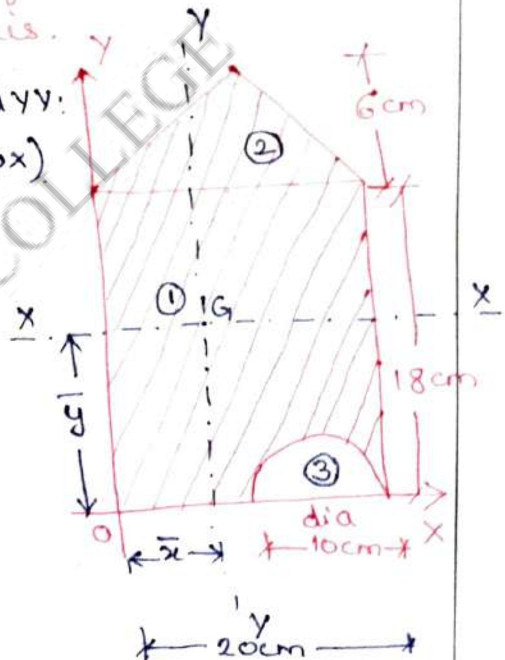
Topic(s) to be covered	

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Determine the moment of inertia of the section shown about its horizontal centroidal axis.



To locate the centroidal axes XX and YY:

Portion (1): Rectangle (Reference OY box)

$a_1 = 20 \times 18 = 360\ cm^2$

$x_1 = \frac{20}{2} = 10\ cm.$

$y_1 = \frac{18}{2} = 9\ cm.$

Portion (2): Triangle

$a_2 = \frac{1}{2} \times 20 \times 6 = 60\ cm^2$

$x_2 = \frac{20}{2} = 10\ cm.$

$y_2 = 18 + \left(\frac{6}{3}\right) = 20\ cm.$

Portion (3): semi-circle.

$a_3 = \frac{1}{2} \times \frac{\pi}{4} \times 10^2 = 39.27\ cm^2.$

$x_3 = 10 + \left(\frac{10}{2}\right) = 15\ cm.$

$y_3 = \frac{4r}{3\pi} = \frac{4 \times 5}{3\pi} = 2.122\ cm.$

$\bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3}{a_1 + a_2 - a_3}$;

$= \frac{(360 \times 10) + (60 \times 10) - (39.27 \times 15)}{360 + 60 - 39.27.}$

$\boxed{\bar{x} = 9.484\ cm.}$

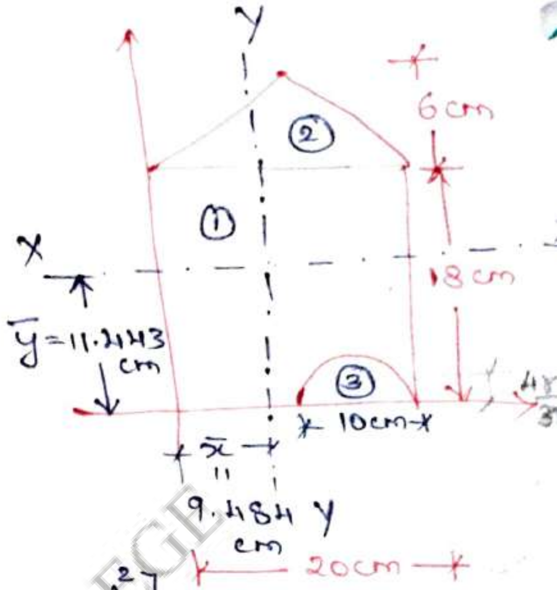
$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3}$

$= \frac{(360 \times 9) + (60 \times 20) - (39.27 \times 2.122)}{360 + 60 - 39.27.}$

$\boxed{\bar{y} = 11.443\ cm.}$

Moment of Inertia about xx-axis:

Moment of inertia of section (1):



$$I_1 = I_{G_1} + A_1\bar{h}_1^2$$

$$= \frac{20\times18^3}{12} + \left[(20\times18)(11.44-9)^2\right]$$

$$= 1.186\times10^4\ cm^4$$

$$I_2 = I_{G_2} + A_2\bar{h}_2^2$$

$$= \frac{bd^3}{36} + A_2\bar{h}_2^2$$

$$= \frac{20\times6^3}{36} + \left[\left(\frac{1}{2}\times20\times6\right)(20-11.4443)^2\right]$$

$$= 4516.41\ cm^4.$$

$$I_3 = I_{G_3} + A_3\bar{h}_3^2$$

$$\frac{4r}{3\pi} = \frac{4\times5}{3\times3.14} = 2.122$$

$$= 0.0068d^4 + A_3\bar{h}_3^2$$

$$= 0.0068(10)^4 + \left[\frac{\pi(5)^2}{2}\times(11.443-2.122)^2\right]$$

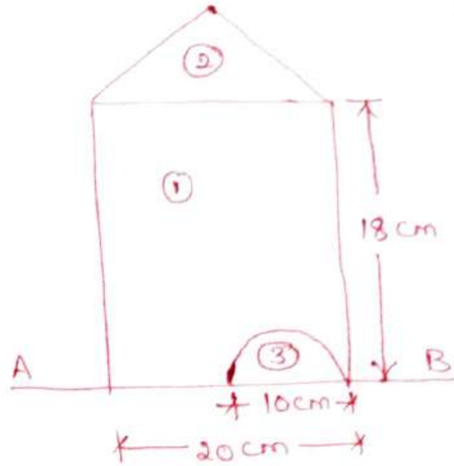
$$= 3478.36\ cm^4$$

$$\therefore I_{xx} = I_1 + I_2 - I_3$$

$$= (1.186\times10^4) + (4516.41) - (3478.36)$$

$$\boxed{I_{xx} = 12898\ cm^4.}$$

In the above problem, calculate the moment of inertia about its bottom edge.



$I_1 = I_{G1} + A_1 \bar{h}_1^2$

$= \frac{20 \times 18^3}{12} + \left[(20 \times 18) \times 9^2 \right]$

$= 38880 \text{ cm}^4.$

$I_2 = I_{G2} + A_2 \bar{h}_2^2$

$= \frac{20 \times 6^3}{36} + \left[\frac{20 \times 6}{2} \times (20)^2 \right]$

$= 24120 \text{ cm}^4.$

$I_3 = I_{G3} + A_3 \bar{h}_3^2$

$= 0.0068 d^4 + \left[\frac{\pi r^2}{2} \times \bar{h}_3^2 \right]$

$= 0.0068 (10)^4 + \left[\frac{\pi (5)^2}{2} \times 2.122^2 \right]$

$= 245.58 \text{ cm}^4.$

$\bar{h}_3 = \frac{4r}{3\pi}$

$= \frac{4 \times 5}{3\pi} = 2.122 \text{ cm}$

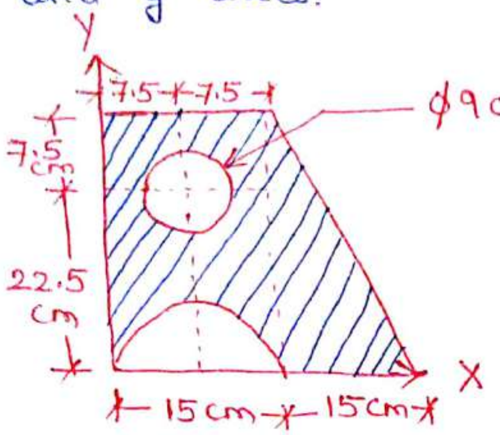
$\therefore I_{AB} = I_1 + I_2 - I_3$

$= 38880 + 24120 - 245.58$

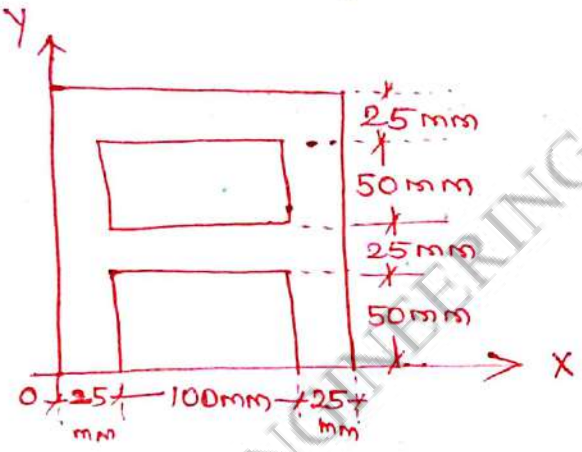
$= 62754 \text{ cm}^4.$

UNIT III - Home Work. (Part A)

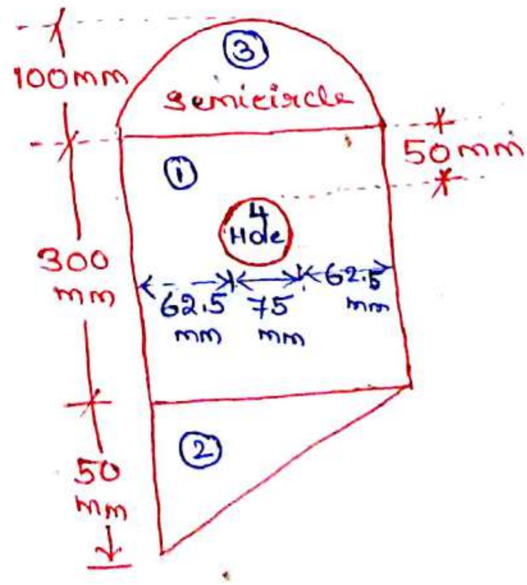
① Find the centroid of the shaded area shown in figure about x and y axes.



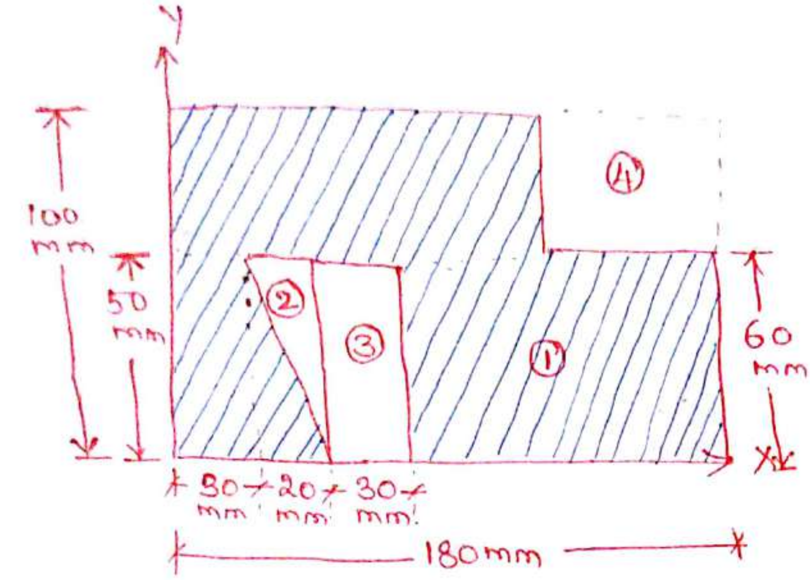
② Locate the centroid of the sections shown.



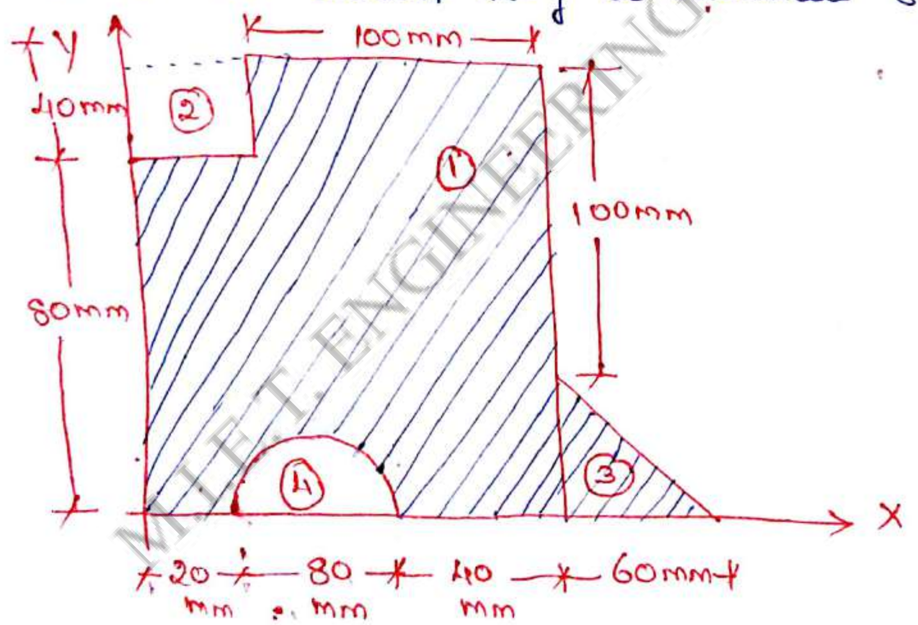
③ For the plane area below, locate the centroid of the area.



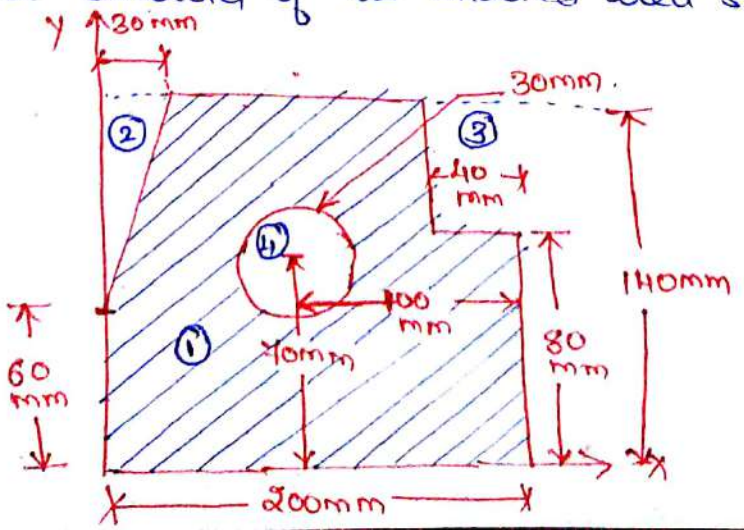
(4) Determine the co-ordinates of the centroid of the shaded area shown below.



(5) Determine the centroidal co-ordinates of the area shown below w.r.t. the shown x-y co-ordinate system.

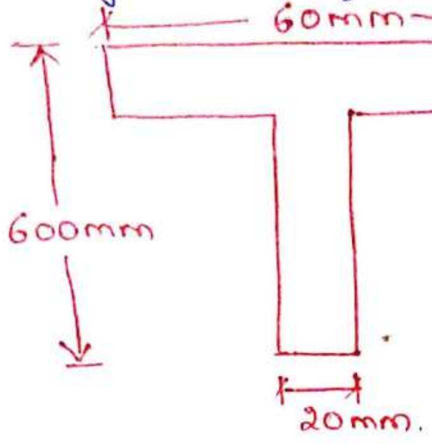


(6) Find the centroid of the shaded area shown below.

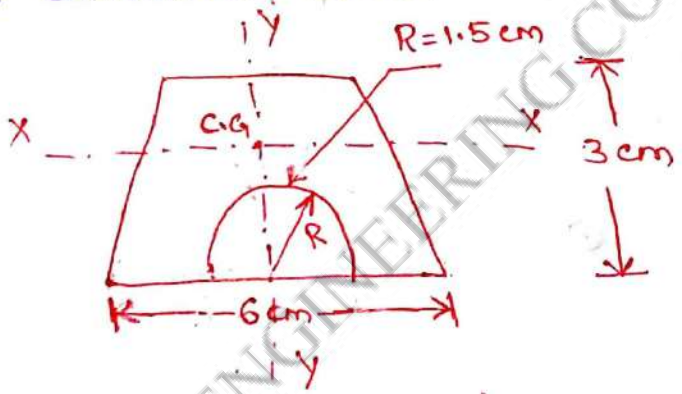


Part B

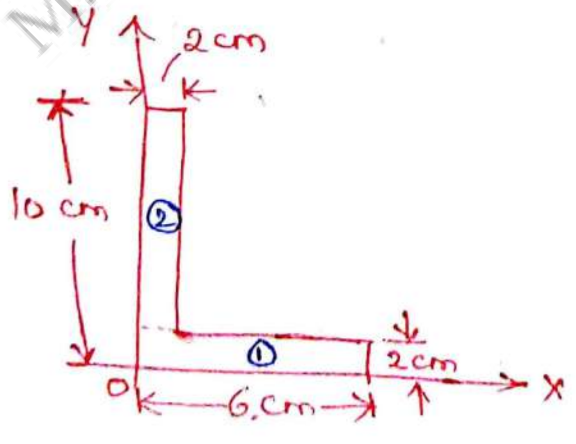
① Calculate the moment of inertia about the horizontal and vertical gravity axes of the section shown in figure



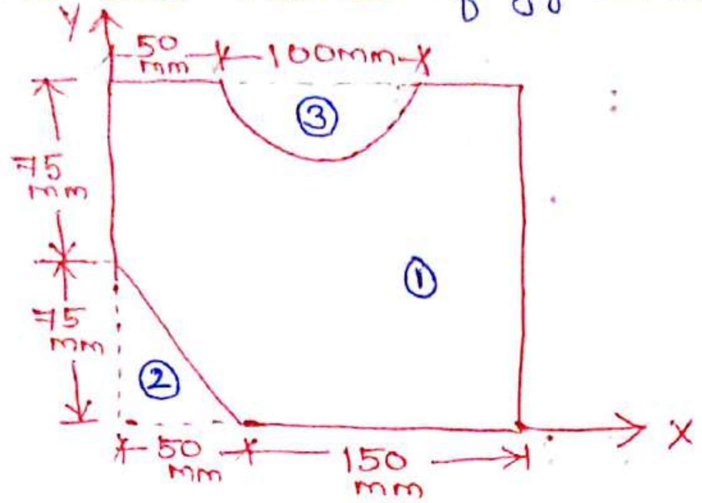
② Find second moment of area of the shaded section shown about its centroidal axes.



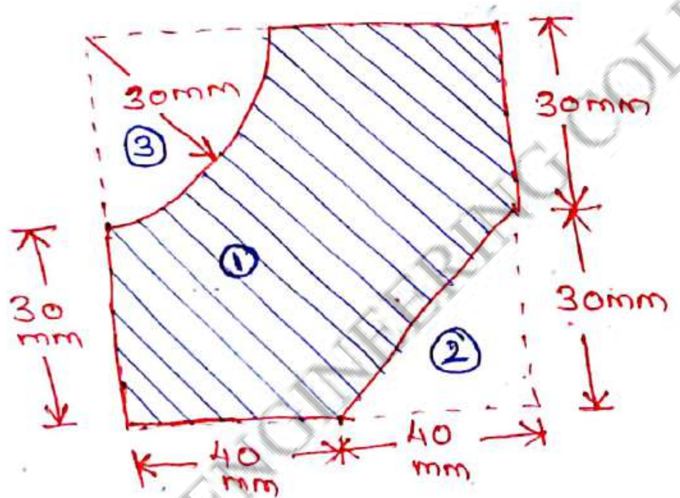
③ Calculate the moment of inertia of L-section shown below about the horizontal axis passing through the C.G.



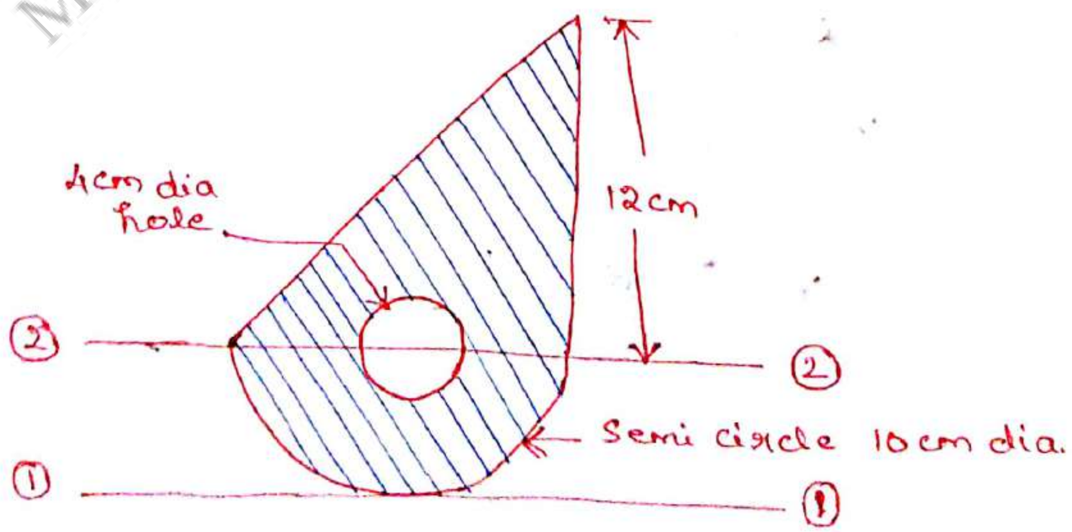
(4) For the plane area shown below determine the area moment of inertia and radius of gyration about the x-axis



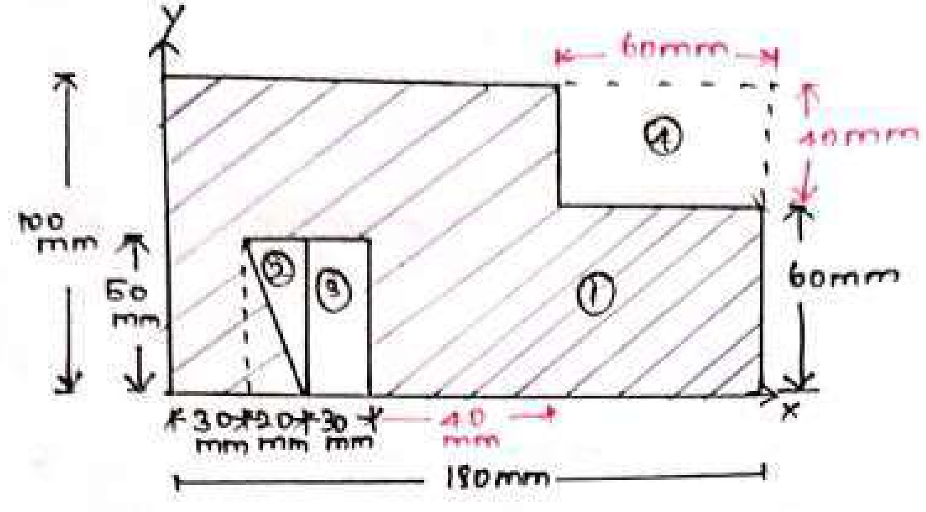
(5) Calculate the moment of inertia and radius of Gyration about x-axis for the sectioned area shown below.



(6) Find the moment of inertia about 1-1 and 2-2 axes for the area shown in figure below:



Determine the co-ordinates of the centroid of the shaded area shown below.



Reference: ox for $\bar{y}$ and oy for $\bar{x}$

Portion ① : (Rectangle)

$a_1 = 100 \times 180 = 18000 mm^2$

$x_1 = \frac{180}{2} = 90mm$

$y_1 = \frac{100}{2} = 50mm$

Portion ② (Triangle)

$a_2 = \frac{1}{2} \times 20 \times 50 = 500 mm^2$

$x_2 = 30 + \frac{2}{3}(20) = 43.33mm$

$y_2 = \frac{2}{3}(50) = 33.33mm$

Portion ③ (Rectangle)

$a_3 = 30 \times 50 = 1500 mm^2$

$x_3 = 30 + 20 + \frac{30}{2} = 65mm$

$y_3 = \frac{50}{2} = 25mm$

Portion ④ (Rectangle)

$a_4 = 60 \times 40 = 2400 mm^2$

$x_4 = 120 + \frac{60}{2} = 150mm$

$y_4 = 60 + \frac{40}{2} = 80mm$

$$\bar{x} = \frac{a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4}{a_1 - a_2 - a_3 - a_4}$$

$$= \frac{(1800 \times 90) - (500 \times 43.33) - (1500 \times 65) - (2400 \times 150)}{1800 - 500 - 1500 - 2400}$$

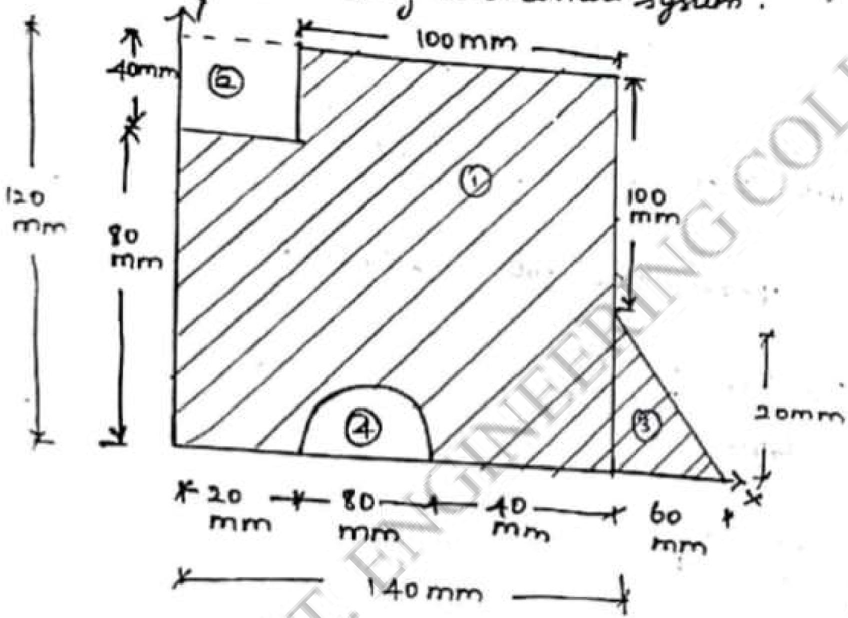
$$\boxed{\bar{x} = 121.98}$$

$$\bar{y} = \frac{a_1y_1 - a_2y_2 - a_3y_3 - a_4y_4}{a_1 - a_2 - a_3 - a_4}$$

$$= \frac{(18000 \times 50) - (500 \times 33.33) - (1500 \times 25) - (2400 \times 80)}{18000 - 500 - 1500 - 2400}$$

$$\boxed{\bar{y} = 48.076}$$

Determine the centroidal co-ordinate of area shown below w.r.t the shown x-y co-ordinate system.



* Assume

* Question

Reference ox for $\bar{y}$ and oy for $\bar{x}$

Portion ① (Rectangle)

$a_1 = 140 \times 120 = 16800\,mm^2$

$x_1 = \frac{140}{2} = 70mm$

$y_1 = \frac{120}{2} = 60mm$

Portion ② (~~Rectangle~~ Square)

$a_2 = \cancel{20}\,40 \times 40 = \cancel{800}\,1600\,mm^2$

$x_2 = \cancel{\frac{20}{2}} = \cancel{10mm}\ \ \frac{40}{2} = 20mm$

$y_2 = \frac{40}{2} + 80 = 100mm$

Portion ③ (Triangle)

$a_3 = \frac{1}{2} \times 60 \times 20 = 600mm^2 \quad x_3 = 140 + \frac{60}{3} = 160mm$

$y_3 = \frac{20}{3} = 6.66mm$

portion ④ (semicircle)

$$a_4 = \frac{\pi r^2}{2} = \frac{\pi \times 40^2}{2} = 2513.27\,mm^2$$

$$x_4 = 20 + \frac{80}{2} = 60\,mm$$

$$y_4 = \frac{4r}{3\pi} = \frac{4 \times 40}{3\pi} = 16.97\,mm$$

$$\bar{x} = \frac{a_1x_1 - a_2x_2 + a_3x_3 - a_4x_4}{a_1 - a_2 + a_3 - a_4}$$

$$= \frac{(16800 \times 70) - \overset{(1600\times20)}{(\cancel{800\times10})} + (600 \times 160) - (2513.27 \times 60)}{16800 - 1600 + 600 - 2513.27}$$

$$= \cancel{\frac{1113203.8}{14086.73}} \qquad = \frac{1089203.8}{13286.73}$$

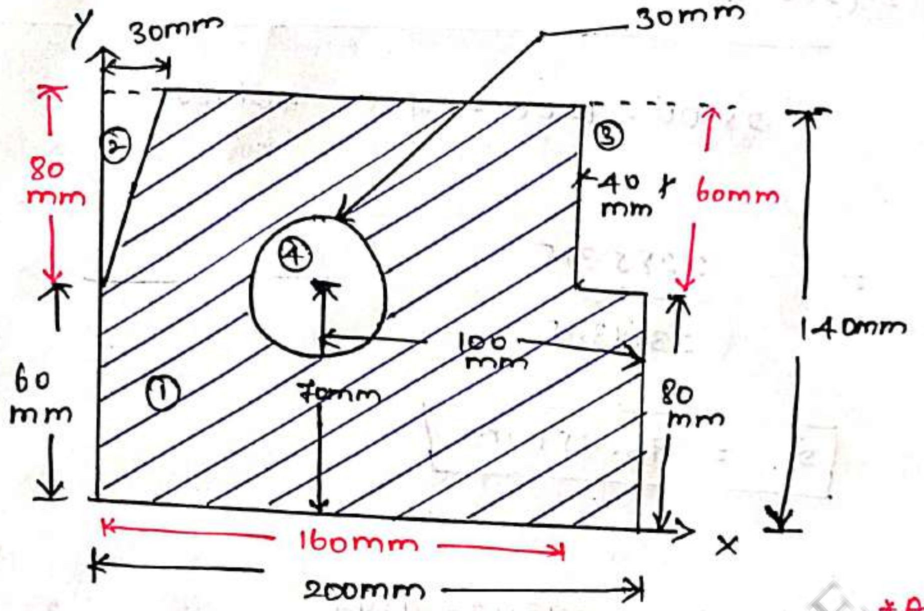
$$\cancel{\boxed{\bar{x} = 79.02\,mm}} \qquad \boxed{\bar{x} = 81.97\,mm}$$

$$\bar{y} = \frac{(16800 \times 60) - (800 \times 100) + (600 \times 6.66) - (2513.27 \times 16.97)}{16800 - 1600 + 600 - 2513.27}$$

$$= \cancel{\frac{-1001814.77}{14086.73}} \qquad = \cancel{\frac{889345.80}{14086.73}} \qquad = \frac{889345.80}{13286.73}$$

$$\cancel{\boxed{\bar{y} = 63.13\,mm}} \qquad \boxed{\bar{y} = 66.93\,mm}$$

Find the centroid of the shaded area shown below:



*Assume Question

Reference OX for $\bar{y}$ and OY for $\bar{x}$

portion ① Rectangle

$a_1 = 200 \times 140 = 28000\,mm^2$

$x_1 = \frac{200}{2} = 100mm$

$y_1 = \frac{140}{2} = 70mm$

portion ② (Triangle)

$a_2 = \frac{1}{2} \times 30 \times 80 = 1200mm^2$

$x_2 = \frac{30}{3} = 10mm$

$y_2 = 60 + \frac{2}{3}(80) = 113.33\,mm$

portion ③ (Rectangle)

$a_3 = 40 \times 60 = 2400mm^2$

$x_4 = 160 + \frac{40}{2} = 180mm$

$y_4 = 80 + \frac{60}{2} = 110mm$

portion ④ (Circle)

$a_4 = \frac{\pi d^2}{4} = \frac{\pi (30)^2}{4}$

$= 706.85\,mm^2$

$x_4 = \frac{200}{2} = 100mm$

$y_4 = 70mm$

$$\bar{x} = \frac{a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4}{a_1 - a_2 - a_3 - a_4}$$

$$= \frac{(28000 \times 100) - (1200 \times 10) - (2400 \times 180) - (706.85 \times 100)}{28000 - 1200 - 2400 - 706.85}$$

$$= \frac{2285315}{23693.15}$$

$$\boxed{\bar{x} = 96.45 mm}$$

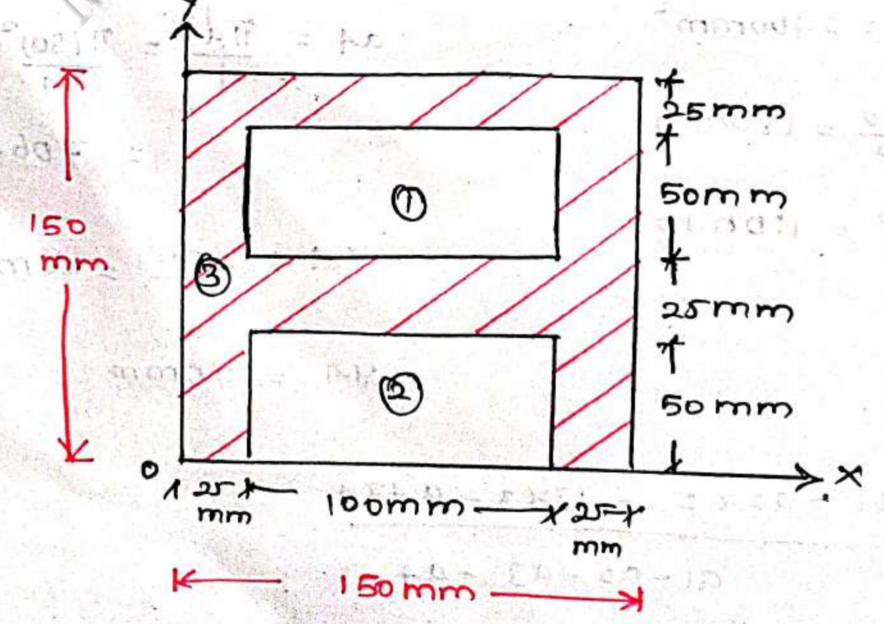
$$\bar{y} = \frac{a_1y_1 - a_2y_2 - a_3y_3 - a_4y_4}{a_1 - a_2 - a_3 - a_4}$$

$$= \frac{(28000 \times 70) - (1200 \times 113.33) - (2400 \times 110) - (706.85 \times 70)}{28000 - 1200 - 2400 - 706.85}$$

$$= \frac{1510524.5}{23693.15}$$

$$\boxed{\bar{y} = 63.75 mm}$$

Locate the centroid of the section shown.



* Assume

* Question

Reference: OX for $\bar{y}$ & OY for $\bar{x}$

due to symmetry about OX axis.

$$\bar{x} = \frac{150}{2} = 75mm$$

$$\boxed{\bar{x} = 75mm}$$

Portion ① (Rectangle)

$a_1 = 100 \times 50 = 5000 mm^2$

$y_1 = 50+25+\frac{50}{2} = 100mm$

Portion ② (Rectangle)

$a_2 = 100 \times 50 = 5000 mm$

$y_2 = \frac{50}{2} = 25mm$

portion ③ (square)

$a_3 = 150 \times 150 = 22500 mm^2$

$y_3 = \frac{150}{2} = 75mm$

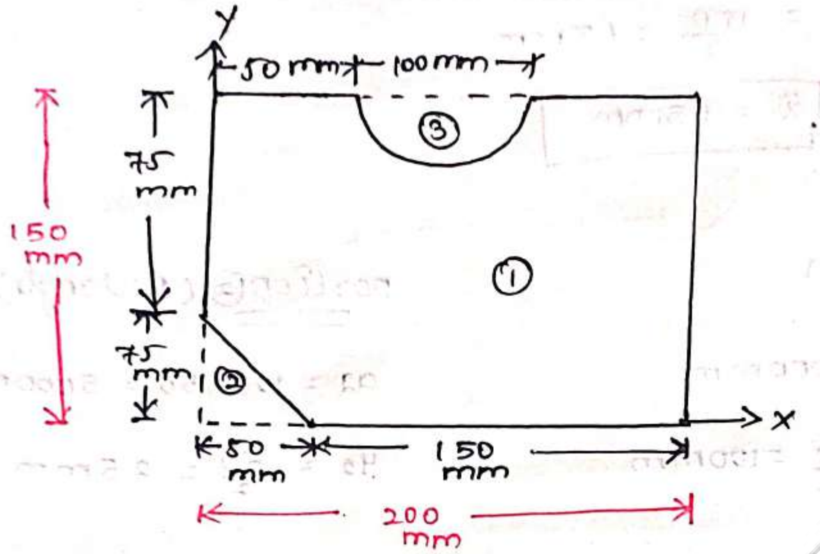
$$\bar{y} = \frac{-(a_1 y_1) - (a_2 y_2) + (a_3 y_3)}{-a_1 - a_2 + a_3}$$

$$= \frac{-(5000 \times 100) - (5000 \times 25) + (22500 \times 75)}{-5000 - 5000 + 22500}$$

$$= \frac{1062500}{12500}$$

$$\boxed{\bar{y} = 85mm}$$

For the plane area below determine the area moment of inertia and radius of gyration about x-axis.



* Assume
* Question

Here, the moment of Inertia about centroidal axis are not required. So, centroid is not required

$M.I = I_1 - I_2 - I_3$

[∴ By parallel Axis theorem $I_{AB} = I_G + Ah^2$

I_1 [portion ① Rectangle]

$I_1 = I_{G1} + A_1 \bar{h}_1^2$

$= \frac{bd^3}{12} + (200 \times 150)\left(\frac{150}{2}\right)$

$= \frac{200 \times 150^3}{12} (30000)(75)$

$I_1 = 225 \times 10^6 \text{ mm}^4$

I_2 [portion ② Triangle]

$I_2 = I_{G2} + A_2 \bar{h}_2^2$

$$= \frac{bh^3}{36} + \left(\frac{1}{2}\times 50\times 75\right)\left(\frac{75}{3}\right)^2$$

$$= \frac{50\times 75^3}{36} + (1875)(25)^2$$

$$= 585937 + [1875\times(25)^2]$$

$$\therefore I_2 = 1757812 \text{ mm}^4$$

I_3 [portion (3) semi-circle] $\left[\begin{array}{l}\therefore d=100 \\ r=50\end{array}\right.$

$$I_3 = I_{G3} + A_3\bar{h}_3^2$$

$$I_3 = 0.0068d^4 + \left[\frac{1}{2}\pi r^2\right]\left[150 - \left[\frac{4r}{3\pi}\right]\right]$$

$$= 0.0068\times 100^4 + \left[\frac{1}{2}\pi\times 50^2\right]\left[150 - \left[\frac{4\times 50}{3\pi}\right]\right]$$

$$= 68\times 10^4 + (3927)(128.78)$$

$$I_3 = 65806500 \text{ mm}^4$$

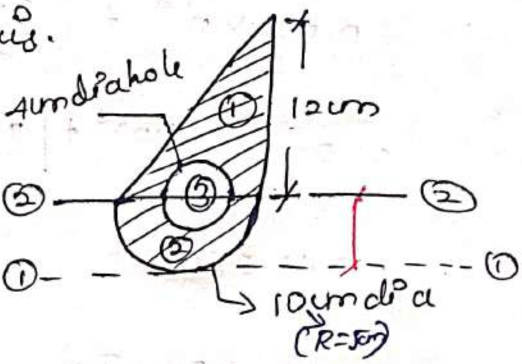
$$M.I_{xx} = I_1 - I_2 - I_3$$

$$= 225\times 10^6 - 1757812 - 65806500$$

$$= 157435688 \text{ mm}^4$$

20/11/2023

For the section shown in figure. Determine the M.O.I values about 1,1 and 2,2 axis.



Sol

$$M.I = M.I_1 + M.I_2 + M.I_3$$

M.O.I about ① – ① axis

portion ① : Traingle :

$$M.I_1 = I_{G1} + A_1 h_1^2$$

$$M.I_1 = \frac{bh^3}{36} + \left[b_1 \times h_1 \times \frac{1}{2} (4+5)^2 \right.$$

$$= \frac{10 \times (12)^3}{36} + \left[10 \times 12 \times \frac{1}{2} \times (9)^2 \right]$$

$$= 480 + 4860$$

$$\boxed{M.I_1 = 5340 cm^4}$$

portion ② : semicircle :

$$M.I_2 = I_{G2} + A_2 h_2^2$$

$\left[\because \text{Area of semicircle} \right\} = \frac{\pi r^2}{2}$

$$= 0.0068 d^4 + \frac{\pi r^2}{2} h_2^2$$

$\because$ centroid of semi circle $= \frac{4r}{3\pi}$ from st. edge

$$= 0.0068 (10)^4 + \left[\frac{\pi \times 5^2}{2} \times \left[r - \frac{4r}{3\pi} \right]^2 \right]$$

$$= 0.0068 (10)^4 + \left[\frac{\pi \times 5^2}{2} \times \left[5 - \frac{4 \times 5}{3\pi} \right] \right]$$

$$= 0.068 \times 10{,}000 + \left[\frac{\pi \times 25}{2} [2.878] \right]$$

$$= 68 + [39.26 \times 8.2828] = 393.18 cm^4$$

portion – ③ circle :

$M.I_3 = I_{G3} + A_3 h_3^2$

$= \frac{\pi(4)^4}{64} + \left[\frac{\pi d^2}{4} \times h_3^2\right]$

$= \frac{\pi \times (4)^4}{64} + \left[\frac{\pi \times (4)^2}{4} \times 5^2\right]$

$= 12.56 + 314.15$

$\left[\therefore \text{Area of circle} = \frac{\pi d^2}{4}\right.$

$h = 5$ cm

Centroid of circle

$M.I = M.I_1 + M.I_2 + M.I_3$

$= 5340 + 393.18 - 326.71$

$\boxed{M.I = 5406.47 \text{ cm}^4}$

Moment of inertia about ② – ②

$M.I = M.I_1 + M.I_2 + M.I_3$

The ② – ② line is on the base so . I_G is not required.

$M.I_1 = I_{AB}$

$= \frac{bd^3}{12}$

$= \frac{10 \times 12^3}{12}$

$= 1440 \text{ cm}^4$

$M.I_2 = I_{AB}$

$= \frac{\pi \times d^4}{64} \times \frac{1}{2}$

$= \frac{\pi \times (10)^4}{64} \times \frac{1}{2}$

$= 245.43 \text{ cm}^4$

$I_{AB} = \frac{1}{2} \times \frac{\pi d^4}{64}$

$$M.I_3 = I_{G3} + A_3 h_3^2$$

$$= \frac{\pi D^4}{64} + (\pi r^2 \times h_3^2)$$

$$= \frac{\pi (4)^2}{64} + [\pi r^2 \times (0)^2]$$

$$= \frac{\pi (4)^2}{64}$$

$$= 12.56\, cm^4$$

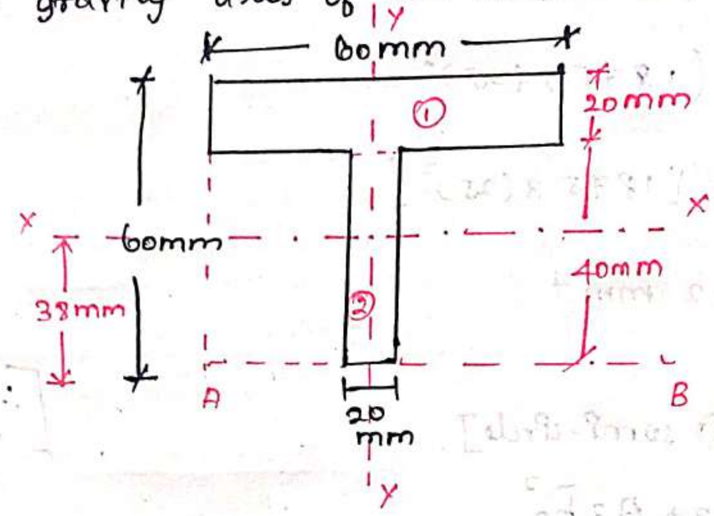
Here: The centroidal axis is passing through ⓧ-ⓧ so $h = 0\,cm$

$$M.I = M.I_1 + M.I_2 - M.I_3$$

$$= 1440 + 245.43 - 12.56$$

$$= 1672.87\, cm^4.$$

Calculate the moment of inertia about the horizontal and vertical gravity axes of the section shown in figure



* Assume
* Question

sol

Reference line ox for $\bar{y}$ & oy for $\bar{x}$

It symmetric about yy axis

$$\boxed{\bar{x} = 30mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

portion① (Rectangle)

$a_1 = b \times h = 60 \times 20 = 1200mm^2$

$y_1 = 40 + \frac{20}{2} = 50mm$

portion② (Rectangle)

$a_2 = 40 \times 20 = 800mm^2$

$y_2 = 20mm$

$$\bar{y} = \frac{(1200)(50) + (800)(20)}{1200 + 800}$$

$$\boxed{\bar{y} = 38mm}$$

M.Ixx = M.I1 + M.I2

$M.I_1 = I_{G1} + A_1\bar{h}_1^2$

$= \frac{bd^3}{12} + (b \times h)(\bar{h}_1^2)$

$h_1 = \bar{y} - (60-10)$

$= 38-50$

$h_1 = 12$

$= \frac{60 \times 20^3}{12} + (60 \times 20)(12^2)$

$= 212800\ mm^4$

$M.I_2 = I_{G2} + A_2\bar{h}_2^2$

$h_2 = \bar{y} - (40)$

$= 38 - 20$

$= 18$

$= \frac{bd^3}{12} + (b \times h)(\bar{h}_2^2)$

$= \frac{20 \times 40^3}{12} + (20 \times 40)(18^2)$

$= 365866.67\ mm^4$

$M.I_{xx} = M.I_1 + M.I_2$

$= 212800 + 365866.67$

$\boxed{M.I_{xx} = 578666.67\ mm^4}$

M.Iyy = M.I1 + M.I2

$M.I_1 = I_{G1} + A_1\bar{h}_1^2$

$h_1 = \frac{60}{2} - \bar{x}$

$= 30 - 30$

$= 0$

$= \frac{db^3}{12} + \cancel{(b \times h)(\bar{h}_1^2)}$

$= \frac{20 \times 60^3}{12}$

$\boxed{M.I_1 = 360000\ mm^4}$

$M.I_2 = I_{G2} + A_2 \bar{h}_2^2$

$h_2 = \bar{x} - 10$

$= 30 - 10$

$= 20$

$= \frac{db^3}{12} + (b \times d)(h_2^2)$

$= \frac{20 \times 40^3}{12} + (40 \times 20)(20^2)$

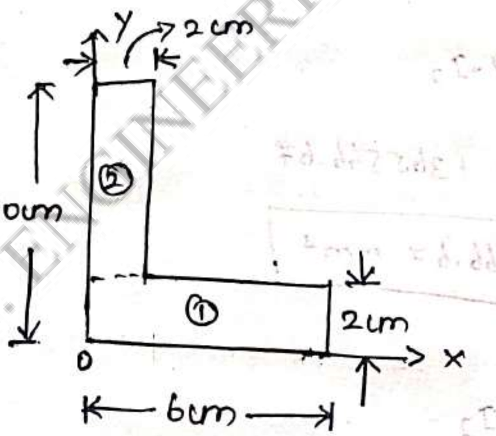
$$\boxed{M.I_2 = 426666.67 \text{ mm}^4}$$

$M.I_{yy} = M.I_1 + M.I_2$

$M.I_{yy} = 360000 + 426666.67$

$$\boxed{M.I_{yy} = 786666.7 \text{ mm}^4}$$

Calculate the moment of inertia of L-section shown below about the horizontal axis passing through the C.G.



Sol

Reference OX for $\bar{y}$ and OY for $\bar{x}$

Portion ① (Rectangle)

$a_1 = b \times h = 6 \times 2 = 12 \text{ cm}^2$

$x_1 = \frac{b}{2} = 3 \text{ cm}$

$y_1 = \frac{2}{2} = 1 \text{ cm}$

section - 2 (Rectangle)

$a_2 = 8 \times 2 = 16\,cm^2$

$x_2 = \frac{2}{2} = 1\,cm$

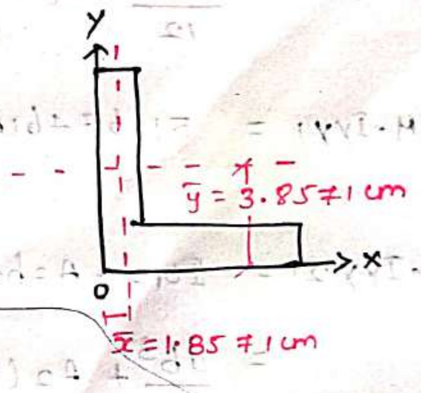
$y_2 = 2 + \frac{8}{2} = 2 + 4 = 6\,cm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(12 \times 3) + (16 \times 1)}{12 + 16}$$

$$\boxed{\bar{x} = 1.8571\,cm}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(12 \times 1) + (16 \times 6)}{12 + 16}$$

$$\boxed{\bar{y} = 3.8571\,cm}$$



M.I = M.Ixx + M.Iyy

M.Ixx = M.Ixx1 + M.Ixx2

$M.I_{xx1} = I_{G1} + A_1\bar{h}_1^2$

$= \frac{bd^3}{12} + A_1(\bar{h}_1)^2$

$h_1 = \bar{y} - y_1$

$= 3.8571 - 1$

$= 2.8571$

$= \frac{6 \times 2^3}{12} + (12)(2.8571)^2$

$M.I_{xx1} = 101.96\,cm^4$

$M.I_{xx2} = I_{G2} + A_2(\bar{h}_2)^2$

$= \frac{bd^3}{12} + A_2(h_2)^2$

$$= \frac{2\times 8^3}{12} + (16)(2.1429)^2$$

$h_2 = \bar{y} \sim y_2$

$= 3.8571 - 6$

$= 2.1429$

$M.I_{xx2} = 158.806$

$M.I_{xx} = M.I_{xx1} + M.I_{xx2}$

$= 101.96 + 158.806$

$$\boxed{M.I_{xx} = 260.76\ cm^4}$$

$M.I_{yy} = M.I_{yy1} + M.I_{yy2}$

$M.I_{yy1} = I_{G1} + A_1 h_1^2$

$$= \frac{db^3}{12} + A_1(\bar{h}_1)^2$$

$h_1 = \bar{x} - x_1$

$= 1.8571 - 3$

$= 1.1429$

$$= \frac{2\times 6^3}{12} + (12)(1.1429)^2$$

$M.I_{yy1} = 51.6746\ cm^4$

$M.I_{yy2} = I_{G2} + A_2 h_2^2$

$$= \frac{db^3}{12} + A_2(h_2)^2$$

$h_2 = \bar{x} \sim x_2$

$= 1.8571 - 1$

$= 0.8571$

$$= \frac{8\times 2^3}{12} + (16)(0.8571)^2$$

$M.I_{yy2} = 17.0872\ cm^4$

$M.I_{yy} = M.I_{yy1} + M.I_{yy2}$

$= \cancel{260}.$ $M.I_{yy} = 51.6746 + 17.0872$

$$\boxed{M.I_{yy} = 68.8\ cm^4}$$

$$M.I = M.I_{xx} + M.I_{yy}$$

$$= 260.76 + 68.8$$

$$\boxed{M.I = 329.56\,\text{cm}^4}$$

Lecture No. 28 **UNIT IV – FRICTION**

Topic(s) to be covered	Friction and types of friction.

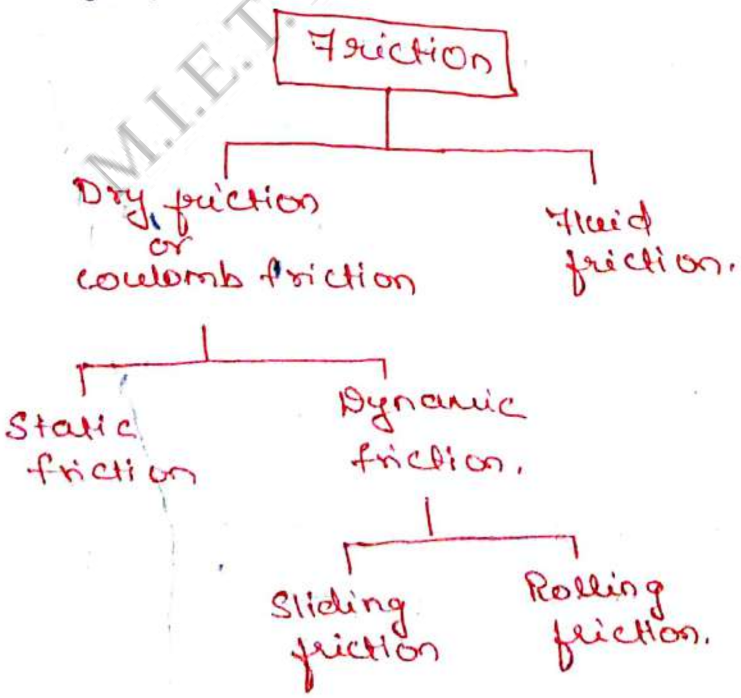
	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	understand various type of friction and laws of friction.	L2

Teaching Learning Material	Student Activity
Chalk and Talk & ppt.	Learn the concept

Lecture Notes

Syllabus: The laws of Dry friction, co-efficients of Friction, Angles of friction, Wedge friction, wheel friction, rolling resistance, Ladder friction.

Friction: In practice, no object is perfectly smooth. When two surfaces are in contact with each other, and one surface tends to move w.r.t. the other, a tangential force will be developed at the contact surface, in the opposite direction of motion. This tangential force is called "Frictional force" or simply "friction".



Dry friction refers to the friction which develops between two dry surfaces slide or tends to slide relative to another.

Fluid friction exists when the contacting surfaces are seperated by a film of fluid.

Static friction is the friction, experienced by a body during rest.

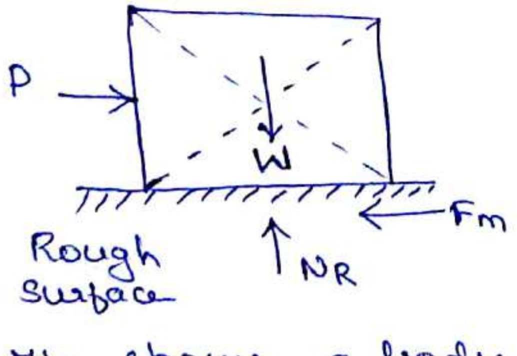
Dynamic or kinetic friction is the friction experienced by a body during motion.

Sliding friction is the friction experienced by a body when it slides over an another.

Rolling friction is the friction experienced by a body when it rolls over a surface.

Limiting friction: The max. resistance offered by the body is called the "Limiting friction". It is denoted by 'F_m'.

$$\text{co-eff. of friction } \mu = \frac{\text{Limiting friction}}{\text{Normal reaction}} = \frac{F_m}{N_R}.$$



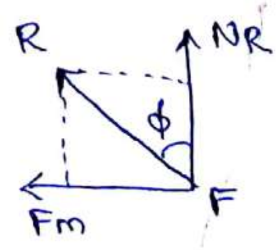


Fig. shows a body of wt. 'W', placed on a rough horizontal surface and subjected to a horizontal push P. When the force exceeds limiting ~~force~~ friction, the body starts moving.

Now, there are two reaction components, 'F_m' acting in the opp. direction of P and normal reaction 'N_R'. These two reactive forces can be combined to a single Resultant "R".

$$R = \sqrt{(N_R)^2 + (F_m)^2}$$

The angle between the resultant 'R' and the normal reaction N_R is called "angle of friction" ϕ

$$\tan\phi = \frac{F_m}{N_R}.$$

$$\Rightarrow \mu = \tan\phi$$

$$F_m = \mu \times N_R.$$

ie Limiting friction = co-eff. of friction x Normal reaction.

co-eff. of friction in static and dynamic are called as co-eff. of static friction (μ_s) and co-eff. of dynamic friction (μ_k) respectively.

$$(F_m)_s = \mu_s \times N_R \text{ and}$$

$$(F_m)_k = \mu_k \times N_R.$$

From experiments, it was found that, co-eff. of kinetic friction is approx. 25% less than that of co-eff. of static friction.

Coulomb's Laws of dry friction.

i) Law of static friction:

1. The frictional force always acts in the opp. direction to that the body tends to move.
2. The frictional force does not depend on the shape and area of contact of the bodies.
3. The frictional force depends on the degree of roughness of the contact area between two bodies.
4. The frictional force is equal to the force applied to the body, so long as the body is in rest.
5. The limiting frictional force (F_m) bears a constant ratio to the normal reaction N_R, between the surface of contact. i.e. $F_m \propto N_R$

$$\Rightarrow F_m = \mu_s N_R.$$

ii) Laws of Dynamic friction:

1. The frictional force always acts in the opp. direction to that the body moves.
2. The magnitude of dynamic friction bears a constant ratio to the normal reaction between the two surfaces.
3. co-eff. of kinetic friction is less than the co-eff. of static friction.

Suggested Questions / Assignments / Home works / any other
copy all definitions in note book.

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press
Any other suggested Materials			
Class notes and Handouts			

Lecture No. 29

UNIT IV – FRICTION

Topic(s) to be covered	Impending Motion.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	understand the basic concepts behind impending motion.	L2

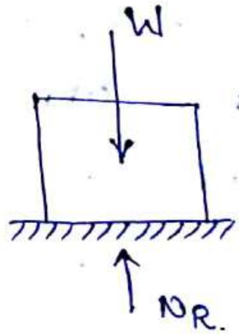
Teaching Learning Material	Student Activity
Power point Presentation.	Listen and learn.

Lecture Notes

Impending motion: The state of motion of a body which is just about to move or slide is called Impending Motion. When the max. frictional force (ie Limiting friction) is attained and if the applied force exceeds the limiting friction, the body starts sliding or rolling. This state is called Impending motion.

Basic concepts:

case (i) $F=0$



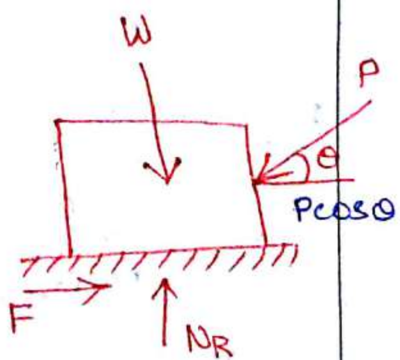
Block of weight W, placed on a smooth horizontal surface. The body is in the condition of equilibrium.

hence to satisfy $\Sigma V=0$,

$$N_R = W.$$

There is no external tangential force, hence friction force $F=0$.

case (ii) $F < F_m$



When force P is inclined at θ

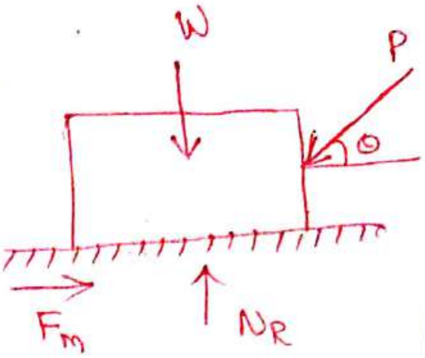
$\Sigma H=0 \Rightarrow F = P\cos\theta.$

$\Sigma V=0 \Rightarrow N_R = W + P\sin\theta$

As $F<F_m$, $F_m = \mu N_R$ cannot be used to find the actual frictional force F.

Case (iii) $F = F_m$

When limiting friction is attained, then the block is in impending motion i.e. block just start to move towards left. ∴ Equations of equilibrium as well as $F_m = \mu N$ can be applied.

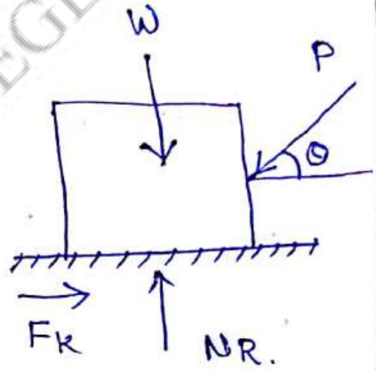


$\Sigma H = 0 \Rightarrow F_m = P\cos\theta$

$\Sigma V = 0 \Rightarrow N_R = W + P\sin\theta$.

Case (iv) $F > F_m$:

If $P\cos\theta$ exceeds limiting friction, then the body is in motion and the body is not in the condition of equilibrium

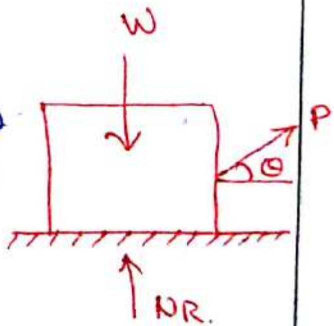


∴ both equations of equilibrium and $F_m = \mu N$ cannot be applied.

But $F_K = \mu_K N$ can be used.

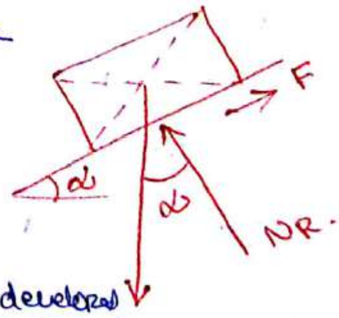
∴ $F_K \neq P\cos\theta$ but $N = W + P\sin\theta$
$N = W - P\sin\theta$.

where μ_K = co-eff. of kinetic friction.



Angle of repose:

The max. inclination of a plane at which a body over that inclined plane remains in equilibrium is called angle of repose.



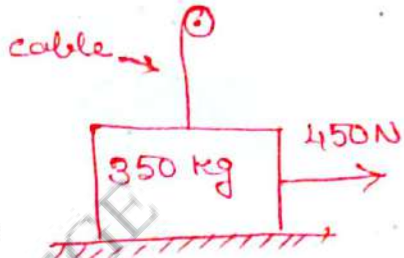
Between contact surfaces, let actual friction developed = F

if $F < \mu_s N_R$, the body is in equilibrium and at rest.

if $F = \mu_s N_R$, motion is impending. i.e about to slide or move

if $F > \mu_s N_R$, body is not in equilibrium and is in motion.

(1) A man can pull horizontally with a force of 450N. A mass of 350 kg is resting on a horizontal surface for which the co-eff. of friction is 0.2. The vertical cable of a crane is attached to the top of the block as shown. What will be the tension in the cable if the man is just able to start the block to the right?

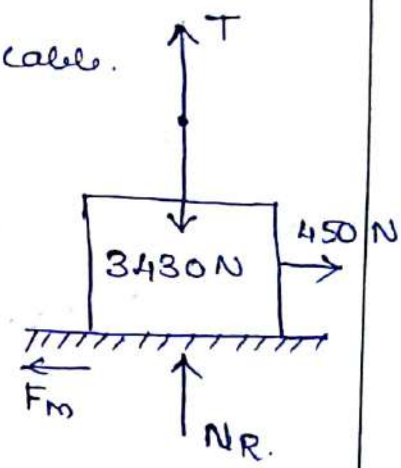


Given, $\mu = 0.2$, mass = 350 kg.

weight of the block = $350 \times 9.81 = 3430N$.

External force P = 450N towards right.

$\therefore$ the frictional force will be developed towards left. Let T be the tension in the cable.



Free Body Diagram

$\Sigma H = 0 \Rightarrow 450 - F_m = 0$

$\boxed{F_m = 450N}$

$\Sigma V = 0 \Rightarrow N_R + T - 3430 = 0$

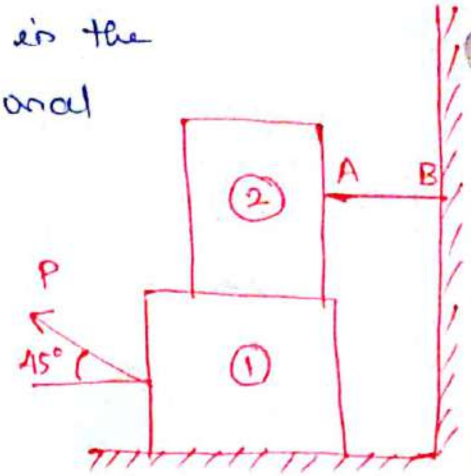
$N_R = 3430 - T$

$\mu = \frac{F_m}{N} \Rightarrow 0.2 = \frac{450}{3430 - T}$

$\boxed{T = 1180N}$ Tension in the cable.

(2) Block (2) rests on block (1) and is attached by a horiz. rope AB to the wall as shown. What force P is necessary to cause motion of block (1) to impend? The co-eff. of friction between the blocks is 1/4 and between the floor and block (1) is 1/3. Mass of blocks (1) and (2) are 14 kg and 9 kg respectively

When the force P pulls block (1), Tension in the cable AB pulls block (2). Hence frictional force in (2) is on L.H.S.



Consider FBD of block (2):

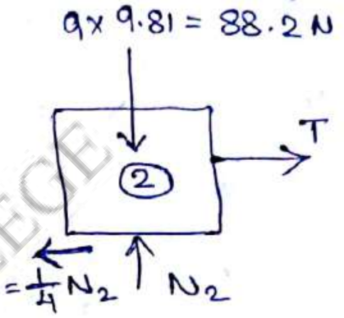
$\Sigma V = 0 \Rightarrow N_2 - 88.2 = 0$

$\boxed{N_2 = 88.2\,N}$

$\therefore F_2 = \mu_2 N_2 = \frac{1}{4} \times 88.2 = 22.05N.$

$\Sigma H = 0 \Rightarrow T - (\mu_2 N_2) = 0.$

$\boxed{T = 0.25 N_2}$ 22.0



FBD of block (2)

Consider FBD of block (1):

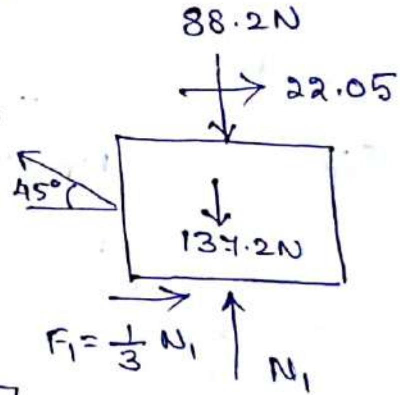
Force P pulls block to left. ∴ frictional force is towards right.

$\Sigma H = 0 \Rightarrow 22.05 + \frac{1}{3}N_1 - P\cos 45^\circ = 0$

$P\cos 45^\circ = 22.05 + 0.333 N_1$ ----- (1)

$\Sigma V = 0 \Rightarrow N_1 + P\sin 45^\circ - 88.2 - 137.2 = 0$

$P\sin 45^\circ = 225.4 - N_1$ ----- (2)



$\frac{\text{eqn. (2)}}{\text{eqn. (1)}} \Rightarrow \frac{P\sin 45^\circ}{P\cos 45^\circ} = \frac{225.4 - N_1}{22.05 + 0.333 N_1}$

$\tan 45^\circ = \frac{225.4 - N_1}{22.05 + 0.333 N_1}$

$\boxed{N_1 = 152.55\,N}$ Sub. in equ. (1), $\boxed{P = 103N.}$

The force reqd. to cause motion in block (1) to impend is 103N.

Suggested Questions / Assignments / Home works / any other
Solve example 1 from text book - P-353

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press
Any other suggested Materials			
Class notes and Handouts			

Lecture No. 30

UNIT IV – FRICTION

Topic(s) to be covered	Friction in inclined plane.

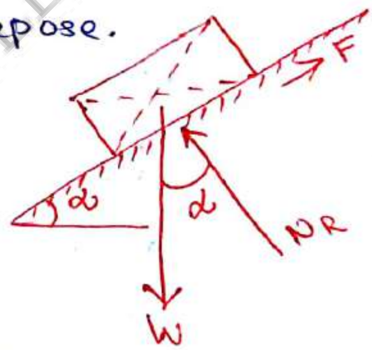
	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO 1	understand the concepts behind friction in inclined plane.	L2

Teaching Learning Material	Student Activity

Lecture Notes

Angle of repose:

The max. inclination of a plane at which a body over that inclined plane remains in equilibrium is called angle of repose.



Between contact surfaces, let actual friction developed = F

If $F < \mu_s N_R$, the body is in equilibrium and at rest.

If $F = \mu_s N_R$, motion is impending. i.e. about to slide or move.

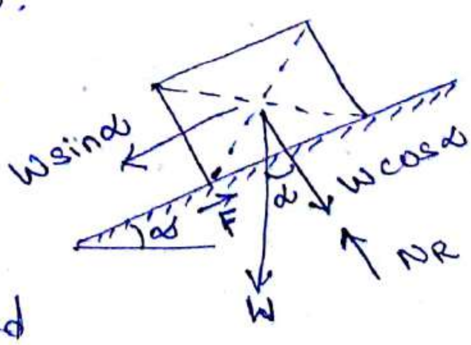
If $F > \mu_s N_R$, body is not in equilibrium and is in motion.

Consider a body of wt. W, resting on an inclined plane at an angle α.

W is resolved into two components, '$W\sin\alpha$' and '$W\cos\alpha$'.

The tagential component $W\sin\alpha$ tends to slide down the block.

and sliding is resisted by upward frictional force F.



The normal reaction $N_R = W\cos\alpha$ and

Frictional force $F = \mu N_R$

$= \mu W\cos\alpha$

when $\mu W \cos\alpha > W \sin\alpha$, the block will remain at rest.

when $\mu W \cos\alpha = W \sin\alpha$, the impending downward motion takes place.

when $W\sin\alpha > \mu W \cos\alpha$, sliding takes place.

Let us denote this angle as 'α_m'

$$\mu W \cos\alpha_m = W \sin\alpha_m$$

$$\mu = \frac{W \sin\alpha_m}{W\cos\alpha_m} = \tan\alpha_m.$$

but we know, $\mu = \tan\phi$.

$$\therefore \alpha_m = \phi.$$

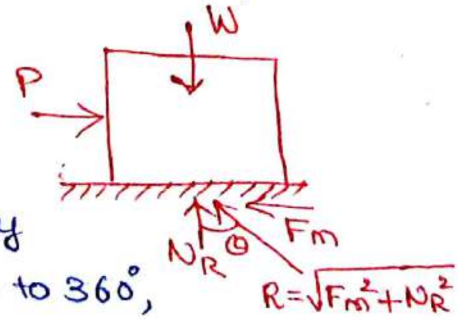
Angle of repose = Angle of static friction.

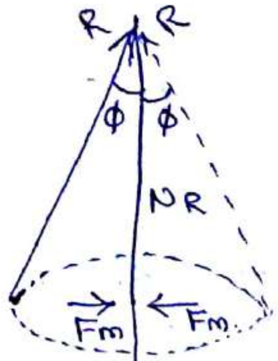
Note: 1. If $\alpha < \phi$, the block will remain at rest and a downward force is to be applied to move the block

2. when $\alpha = \phi$, the body starts sliding.

3. when $\alpha > \phi$, the body will slide down with its own weight alone, without application of any external force.

Cone of friction:





cone of friction.

If a horizontal force P is changed gradually at an angle from $\theta = 0$ to $360°$, then the line of action of resultant reaction R will also be changed. It generates a right circular cone with semi central angle equal to the angle of static friction. This is known as cone of friction.

Lecture No. 31, 32 UNIT IV – FRICTION

Topic(s) to be covered	Friction in inclined plane

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	Solve problems on friction in inclined plane	L3

Teaching Learning Material	Student Activity
Chalk and Talk.	Learn and solve

Lecture Notes

(3) An effort of 200N is required just to move a certain body up an inclined plane of angle 15°, the force acting parallel to the plane. If the angle of inclination of the plane is made 20°, the effort reqd. being again parallel to the plane, is found to be 230N. Find the weight of the body and the co-eff. of friction. P-381

Case 1: Effort reqd. = 200N ; angle of plane = 15°

Case 2: Effort reqd = 230N ; angle of plane = 20°

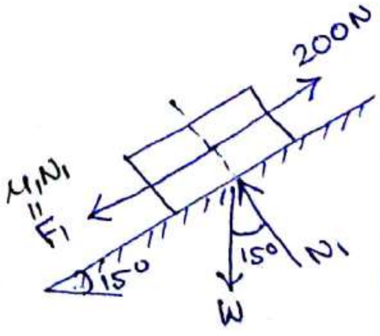
In both cases, the body moves up the inclined plane ∴ frictional force $F = \mu N$ acts downwards.

Case 1: Resolving forces along the plane,

$200 - W\sin 15^\circ - F_1 = 0$

$200 - W\sin 15^\circ - \mu N_1 = 0$

$W\sin 15^\circ + \mu N_1 = 200 \quad \ldots (1)$



Resolving forces normal to the plane,

$N_1 - W\cos 15^\circ = 0$

$N_1 = W\cos 15^\circ \quad \ldots (2)$

Sub. (2) in (1),

$W\sin 15^\circ + \mu (W\cos 15^\circ) = 200.$

$W(\sin 15^\circ + \mu\cos 15^\circ) = 200 \quad \ldots (3)$

case 2: Resolving forces along the plane.

$$230 - W\sin 20 - F_2 = 0$$

$$230 - W\sin 20 - \mu N_2 = 0$$

$$W\sin 20 + \mu N_2 = 230 \quad \text{------ (4)}$$

Resolving forces normal to the plane,

$$N_2 - W\cos 20^\circ = 0$$

$$N_2 = W\cos 20^\circ \quad \text{------ (5)}$$

Sub. N_2 in equ. (4),

$$W\sin 20 + \mu(W\cos 20^\circ) = 230 \quad \text{------ (6)}$$

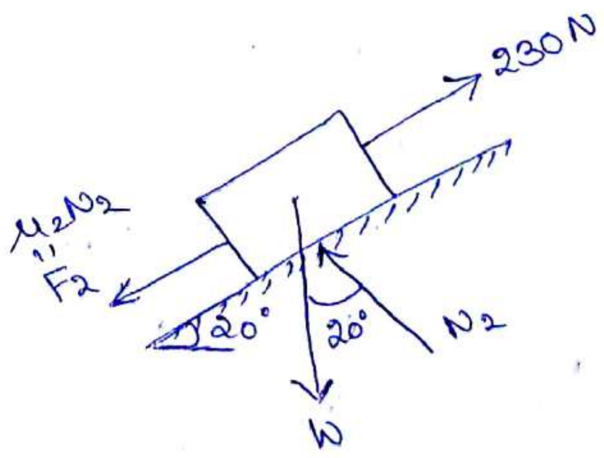
Solving equ. (3) and (6)

$$\frac{\text{equ. (6)}}{\text{equ. (3)}} \Rightarrow \frac{W(\sin 20 + \mu\cos 20)}{W(\sin 15^\circ + \mu\cos 15^\circ)} = \frac{230}{200} = 1.15$$

$$\sin 20 + \mu\cos 20 = 1.15(\sin 15 + \mu\cos 15)$$

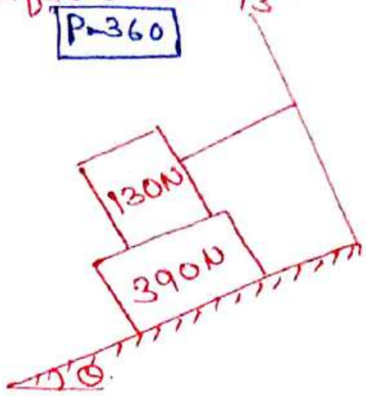
$\boxed{\mu = 0.25}$ co-eff of friction.

$\boxed{W = 392.3\text{ N}}$ weight of the body.



(4) What should be the value of the angle θ so that motion of the 390N block impends down the plane? The co-eff of friction μ for all surfaces is 1/3

P-360



Let T be the tension in the cable.

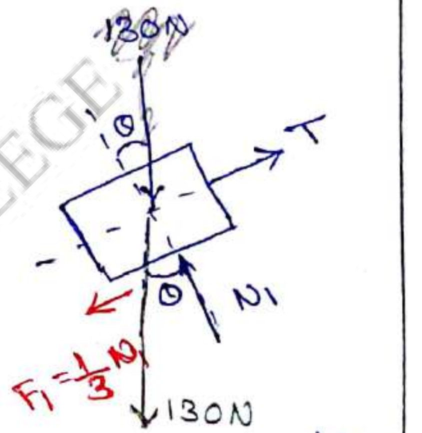
consider FBD of upper block. (a):

Resolving forces along the plane

$T - 130\sin\theta - F_1 = 0$

$T - 130\sin\theta - \mu N_1 = 0$

$T = 130\sin\theta + \frac{1}{3}N_1$ ...... (1)



a) FBD of upper block.

Resolving forces normal to the plane

$N_1 = 130\cos\theta$ .... (2)

Sub. in equ. (1),

$T = 130\sin\theta + \left(\frac{1}{3} \times 130\cos\theta\right)$

$T = 130\sin\theta + 43.33\cos\theta$. ---- (3)

consider FBD of lower block (b):

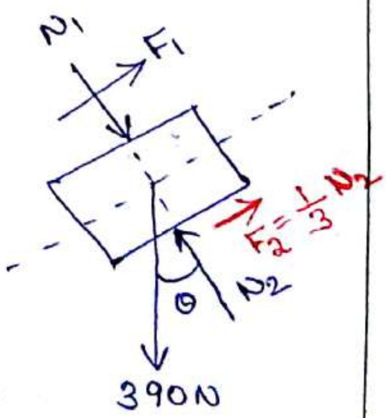
Resolving forces along the plane

$F_1 + F_2 - 390\sin\theta = 0$

$390\sin\theta = F_1 + F_2 = \mu N_1 + \mu N_2$

$= \mu(N_1 + N_2)$

$= \frac{1}{3}(130\cos\theta + N_2)$ ---- (4)



Resolving forces normal to the plane

$N_2 - N_1 - 390\cos\theta = 0$

$N_2 = 390\cos\theta + 130\cos\theta = 520\cos\theta$.

b) FBD of lower block

Sub. N_2 in equ. (4),

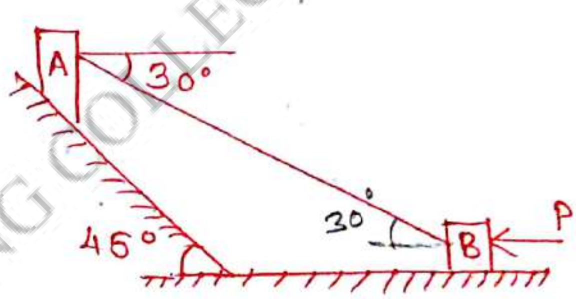
$390\sin\theta = \frac{1}{3}(130\cos\theta + 520\cos\theta)$.

$390\sin\theta = \frac{1}{3}(650\cos\theta)$

$\frac{\sin\theta}{\cos\theta} = \frac{216.67}{390} \Rightarrow \tan\theta = 0.555$

$\boxed{\theta = 29^\circ}$ sub. in (3), $\boxed{T = 100.92N}$

(5) Block A weighing 1000N rests on a rough inclined plane whose inclination to the horizontal is 45°. It is connected to another block B, weighing 3000N rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal as shown. Find the horizontal force reqd to be applied to the block B just to move the block A in upward direction. Assume angle of friction as 15° at all surfaces where there is sliding.



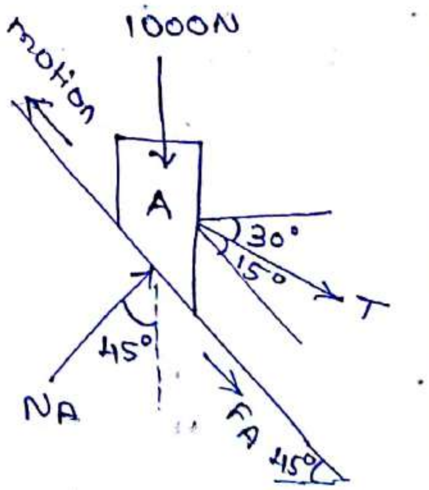
Given $\phi = 15°$. $\mu = \tan\phi = \tan 15°$

$\mu = 0.268$.

Let T = tension in rod, connecting the blocks A and B.

Consider FBD of block A:

Resolve forces normal to the plane and equate to zero.



$N_A - 1000\cos 45° + T\sin 15° = 0$

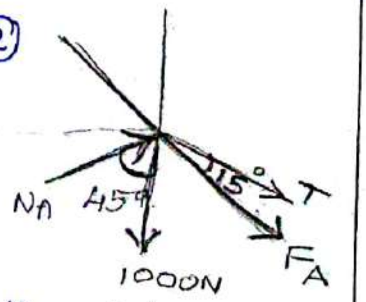
$N_A + 0.259T = 707.1$ ...... (1)

Resolve force along the plane and equate to zero.

$T\cos 15° + F_A + 1000\sin 45° = 0$.

$T\cos 15° + (\mu N_A) + 1000\sin 45° = 0$

$0.966T + 0.268N_A = -707.1$ ...... (2)



solving equ. (1) and (2)

$\boxed{T = -1000N}$ and $\boxed{N_A = 448.1N}$

Negative sign indicates that nature of force in the rod is compressive.

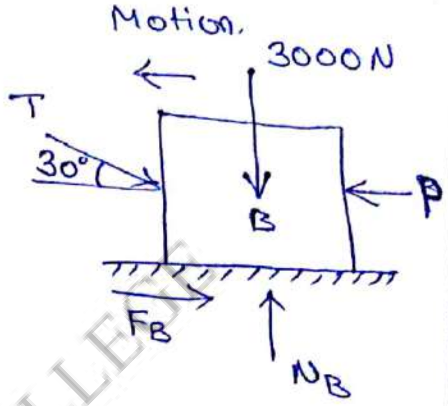
Consider free body diagram of block B.

Applying $\Sigma V = 0$ ($\uparrow$ +ve),

$N_B - 3000 - T \sin 30^\circ = 0$

$N_B - 3000 - (1000 \sin 30^\circ) = 0$

$\boxed{N_B = 3500\,N}$

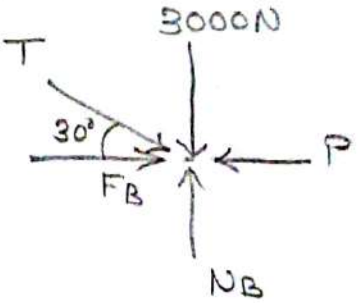


Applying $\Sigma H = 0$

$T \cos 30^\circ + F_B - P = 0.$

$1000 \cos 30^\circ + (0.268 \times 3500) = P.$

$\boxed{P = 1804\,N}$



Suggested Questions / Assignments / Home works / any other
Solve example 4, Pg - 359.

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press
Any other suggested Materials			
Class notes and Handouts			

Lecture No. 33, 34 **UNIT IV – FRICTION**

Topic(s) to be covered	Simple contact friction.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	understand and solve ladder friction and wedge friction.	L3

Teaching Learning Material	Student Activity
chalk and Talk	Learn and Solve.

Lecture Notes

Simple contact friction:

Frictional force is the resisting force developed at the contact surface of two bodies due to their roughness and when the surface of one body moves over the surface of an another body.

They are categorised as:

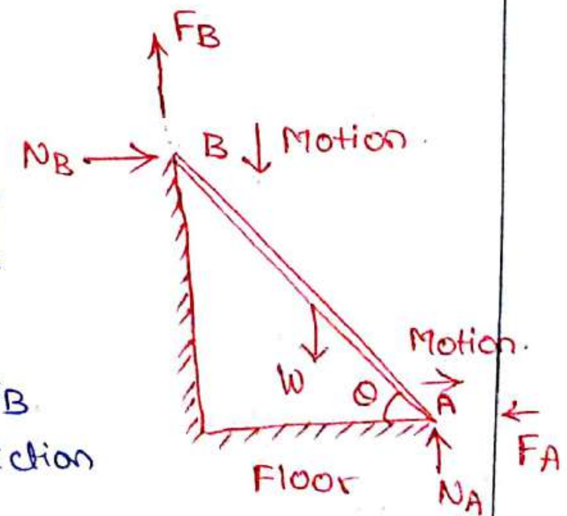
i) Ladder friction.

ii) Wedge friction

iii) Screw friction

iv) Belt friction.

1. Ladder friction:



Consider a ladder AB. During sliding, the upper end of ladder tends to slip downwards and hence the frictional force at B, F_B acts upwards. If co-eff of friction of wall B is μ_B,

then $\mu_B = \frac{F_B}{N_B} \Rightarrow \boxed{F_B = \mu_B N_B}$

$||^{ly}$ end A moves away from wall. ∴ F_A acts towards left.

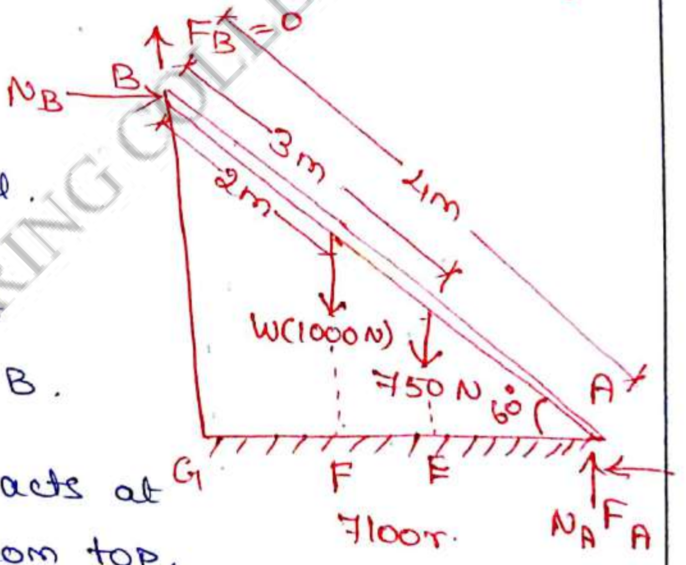
$\mu_A = \frac{F_A}{N_A} \Rightarrow \boxed{F_A = \mu_A N_A}$ μ_A = co-eff. of floor.

For the impending motion of the ladder, the equilibrium eqns. $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$ and $F = \mu N$ are to be satisfied.

Dr.P.V.Premalatha, Prof/CIVIL

Note: Here both ground and wall surfaces are taken as rough. But sometimes ladder is placed against smooth wall where F = 0.

(6) A uniform ladder of weight 1000N and of length 4m rests on a horizontal ground and leans against a smooth vertical wall. The ladder makes an angle of 60° with horizontal. When a man of wt- 750 N stands on the ladder at a distance 3m from the top of the ladder, the ladder is at a pt. of sliding. Determine the co-eff. of friction between the ladder and the floor.



F_A = Frictional force at A.
$= \mu_A N_A$

$F_B = 0$, due to smooth wall.

μ_A = co-eff. of friction at A.

N_A = Normal reaction at A

N_B = Normal reaction at B.

Self wt. of ladder, 1000 N acts at mid pt. of ladder. = 2m from top.

Applying $\Sigma V = 0$ ($\uparrow$ +ve).

$$N_A - 1000 - 750 = 0$$

$$\boxed{N_A = 1750\,N}$$

Applying $\Sigma H = 0$ ($\rightarrow$ +ve).

$$N_B - F_A = 0$$

$$N_B - \mu_A N_A = 0$$

$$N_B = \mu_A N_A = 1750\,\mu_A \quad \ldots\ldots (1)$$

$\Sigma M_A = 0$ (↺ +ve).

$(N_B \times BG) - (1000 \times AF) - (750 \times AE) = 0$

$BG = 4\sin 60^\circ = 3.46m$; $AF = 2\cos 60^\circ = 1m$; $AE = 1\cos 60^\circ = 0.5m$

$(N_B \times 3.464) - (1000 \times 1) - (750 \times 0.5) = 0$

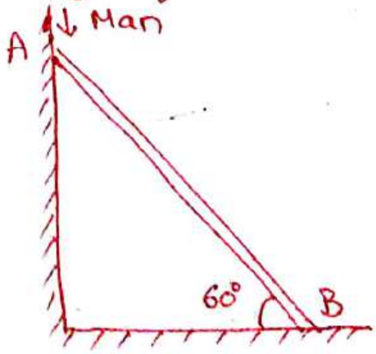
$$\boxed{N_B = 396.9 N.}$$

Sub. ①, $N_A = 1750 \mu_A$. , $N_B = \mu_A \times 1750$

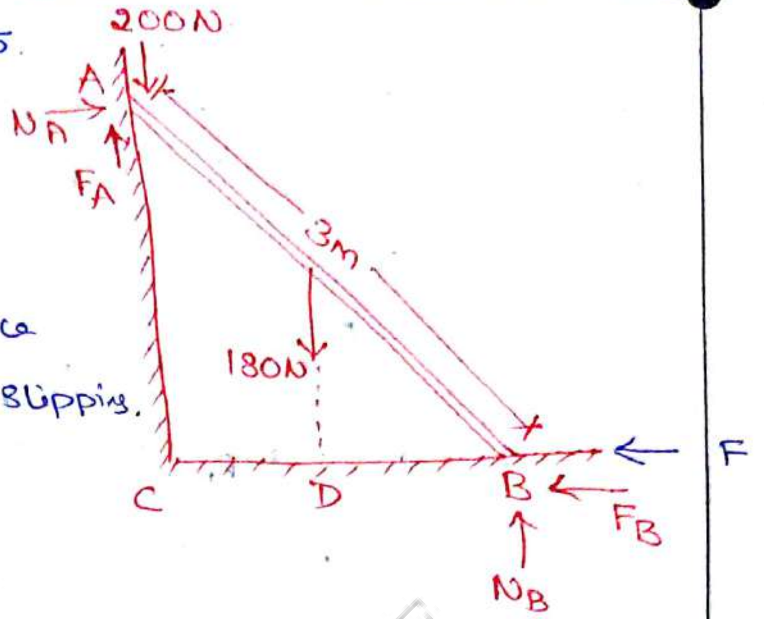
$$\mu_A = \frac{N_B}{1750} = \frac{396.9}{1750} = 0.226.$$

The co-eff. of friction between the ladder and the floor is 0.226.

⑦ A uniform ladder 3m long weighs 180N. It is placed against a wall making an angle 60° with the floor as shown. The co-eff of friction between the wall and the ladder is 0.25 and that between the floor and the ladder is 0.35. The ladder in addition to its own weight, has to support a man weighing 200N at its top at A. Calculate i) the horizontal force F to be applied to the ladder at the floor level to prevent slipping.
ii) If the force 'F' is not applied, what should be the minimum inclination of the ladder with the horizontal so that there is no slipping of it with the man at the top ?



Given $\mu_A = 0.25$; $\mu_B = 0.35$.



case (i) Horizontal force at the floor level to prevent slipping

Let 'F' be the horz. force to be applied to prevent slipping.

Applying $\Sigma H = 0$, ($\rightarrow$ +ve)

$$N_A - F_B - F = 0$$

$$\boxed{F = N_A - F_B}$$

Applying $\Sigma V = 0$, ($\uparrow$ +ve)

$$N_B - 180 - 200 + F_A = 0$$

$$\boxed{N_B = 380 - F_A}$$

Applying $\Sigma M_B = 0$ ($\circlearrowright$ +ve).

$$-(200 \times CB) + (N_A \times AC) + (F_A \times CB) - (180 \times BD) = 0.$$

$$-(200 \times 1.5) + (2.598\,N_A) + (0.25 N_A \times 1.5) - (180 \times 0.75) = 0.$$

$CB = 3\cos 60° = 1.5$

$AC = 3\sin 60° = 2.598$

$BD = \frac{CB}{2} = 0.75.$

$$-300 + 2.598\,N_A + 0.375 N_A - 185 = 0$$

$$2.973\,N_A = 435$$

$$\boxed{N_A = 146.32\,N}$$

$\therefore F_A = \mu_A N_A = 0.25 \times 146.32 = 36.58\,N.$

$N_B = 380 - F_A = 380 - 36.58 = 343.42\,N.$

$F_B = 0.35 \times 343.42 = 120.19\,N.$

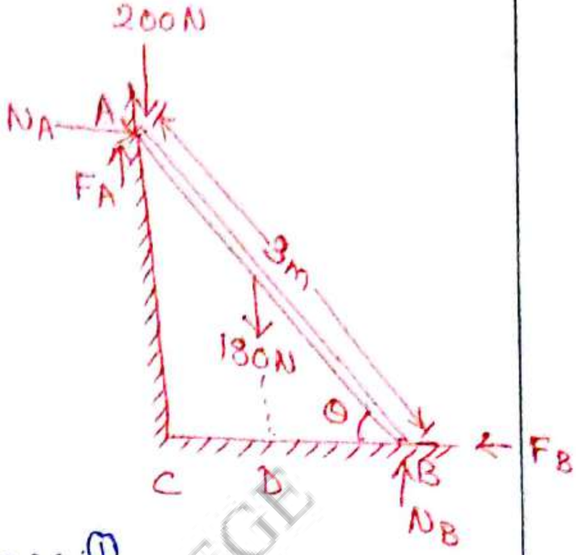
$\therefore F = N_A - F_B = 146.31 - 120.19 = 26.12\,N.$

ii) Minimum angle θ, without the force F, to avoid slipping

Applying $\Sigma H=0$ ($\rightarrow$ +ve).

$N_A - F_B = 0$

$\boxed{N_A = F_B}$



Applying $\Sigma V=0$ ($\uparrow$ +ve).

$F_A + N_B - 200 - 180 = 0$

$F_A + N_B = 380$

$\mu_A N_A + \dfrac{F_B}{\mu_B} = 380$ ---- ①

$0.25 N_A + \dfrac{N_A}{0.35} = 380$. $[\because F_B = N_A]$

$\boxed{N_A = 122.3N}$

Sub. in equ. ②

$\boxed{N_B = 349.42N}$ $F_A = \mu_A N_A = 0.25 \times 122.3$

$\boxed{F_A = 30.575}$

Applying $\Sigma M_B = 0$ (↺ +ve).

$(N_A \times AC) + (F_A \times CB) - (200 \times CB) - (180 \times DB) = 0$

$(122.3 \times 3\sin\theta) + (30.575 \times 3\cos\theta) - (200 \times 3\cos\theta)$

$-(180 \times 1.5\cos\theta) = 0$

$366.9\sin\theta + 91.725 - 600\cos\theta - 270\cos\theta = 0$

$366.9\sin\theta - 778.275\cos\theta = 0$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{778.275}{366.9}$

$\tan\theta = 2.121$

$\theta = \tan^{-1}(2.121)$

$\boxed{\theta = 64.75°}$

$\boxed{\begin{array}{l} AC = 3\sin\theta \\ CB = 3\cos\theta \\ DB = 1.5\cos\theta \end{array}}$

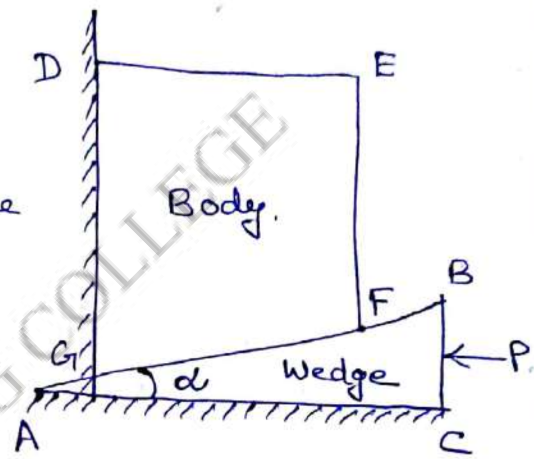
Topic(s) to be covered	Wedge friction.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	understand and apply the concept of wedge friction	L3

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

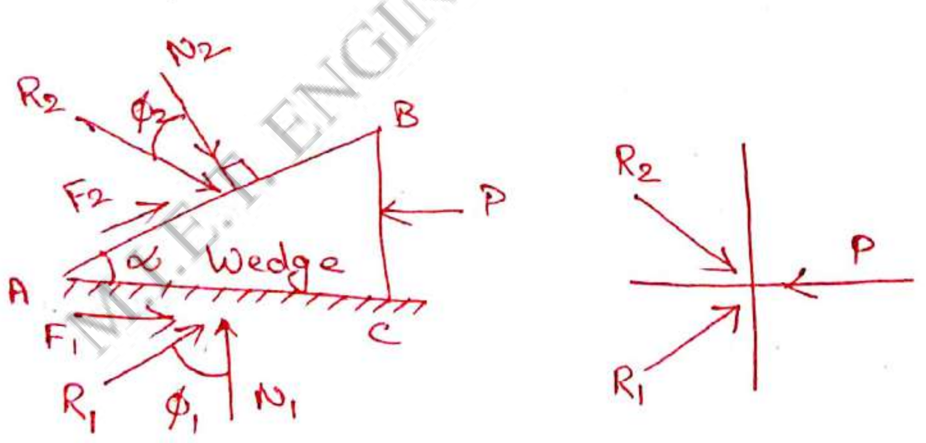
Lecture Notes

Wedge friction:



Consider a wedge, placed under a body to lift upwards as shown. When the force P is pushed to move the wedge towards left, frictional force acts opposite to the direction of force.

Equilibrium of wedge:



When the force P is applied towards left, frictional force F_1 and F_2 will be developed towards right; and the normal reaction N_1 acts upwards and N_2 acts downwards.

$F_2 = \mu_2 N_2$ and $F_1 = \mu_1 N_1$

where μ_1 and μ_2 are co-eff. of friction on the edges AC and AB.

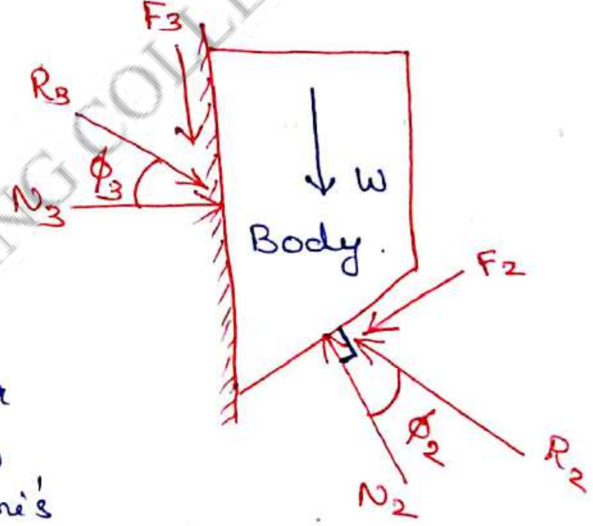
The resultant $R = \sqrt{F^2 + N_R^2}$.

$R_1 = \sqrt{F_1^2 + N_1^2}$ and $R_2 = \sqrt{F_2^2 + N_2^2}$.

F_1 and F_2 are the limiting frictions.

Now, the forces R_1, R_2 and P are the coplanar concurrent forces in equilibrium, which can be analysed by Lami's theorem or by appling equations of equilibrium.

Equilibrium of body:

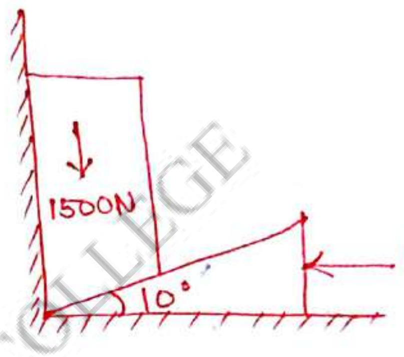


When force P is applied, the body moves upwards. Hence frictional force F_2 and F_3 acts downwards. R_2, R_3 and self wt. of body W are the coplanar concurrent forces in equilibrium which can be analysed by Lami's theorem or by applying equations of equilibrium.

Note:

1. Always draw FBD of wedge first, then the block
2. But, while solving if load is given, solve FBD of block first and if force P is given, solve FBD of wedge first.
3. Self weight of wedge is neglected.

8) A block overlying a 10° wedge on a horz. floor and leaning against a vertical wall and weighing 1500N is to be raised by applying a horz. force to the wedge. Assuming co-eff. of friction between all the surfaces in contact to be 0.3, determine the min. horz. force to be applied to raise the block.



Angle of the wedge = 10°

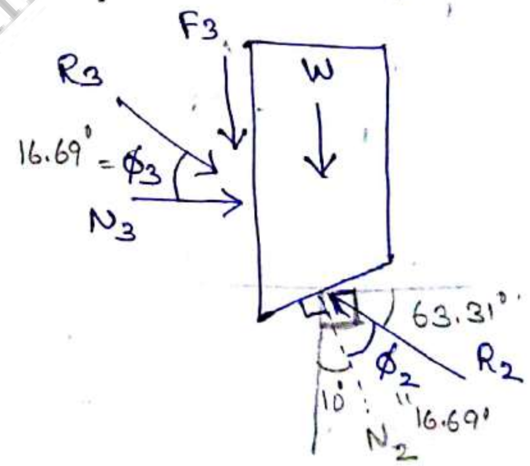
$W = 1500N$, co-eff. of friction $\mu = 0.3$

$\mu = \tan\phi$

$\therefore$ angle of friction $\phi = \tan^{-1}(\mu)$

$\phi = \tan^{-1}(0.3) = 16.69°$

consider FBD of block.



Applying $\Sigma H = 0$ ($\rightarrow$ +ve)

$R_3 \cos 16.69 - R_2 \cos 63.31 = 0$.

$R_3 \cos 16.69 = R_2 \cos 63.31$

$0.9578 R_3 = 0.4491 R_2$.

$\boxed{R_3 = 0.468 R_2}$

Applying $\Sigma V = 0$ ($\uparrow$ +ve).

$-R_3 \sin 16.69 - 1500 + R_2 \sin 63.31 = 0$ ...

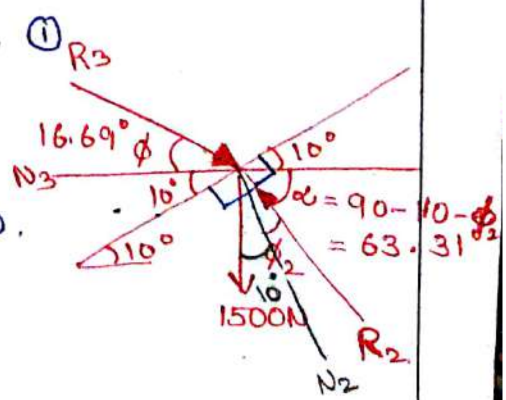
$0.893 R_2 - 0.287 R_3 = 1500$. .... (1)

Sub. $R_3 = 0.468 R_2$ in equ. (1)

$0.893 R_2 - 0.287(0.468 R_2) = 1500$.

$\boxed{R_2 = 1977N}$

$\boxed{R_3 = 925N}$



Consider FBD of wedge

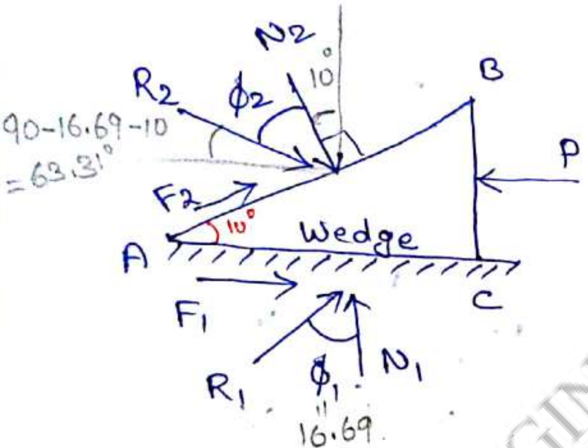
Applying $\Sigma V = 0$ ($\uparrow$ +ve)

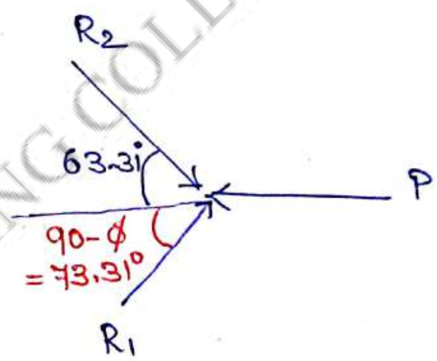
$$R_1 \sin 73.31 - R_2 \sin 63.31 = 0$$

$$R_1 \sin 73.1 = R_2 \sin 63.3$$

$$= 1977 \sin 63.3^\circ$$

$$\boxed{R_1 = 1846\,N}$$





Applying $\Sigma H = 0$ ($\rightarrow$ +ve)

$$R_2 \cos 63.31^\circ + R_1 \cos 73.31 - P = 0$$

$$1977 \cos 63.1^\circ + 1846 \cos 73.1 = P$$

$$\therefore \boxed{P = 1431\,N}$$

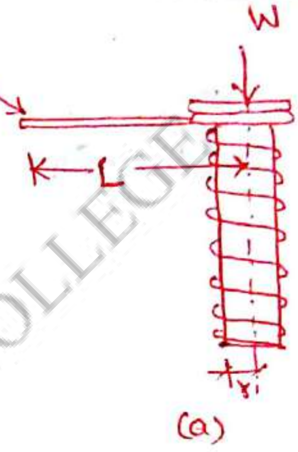
Lecture No. 35 **UNIT IV – FRICTION**

Topic(s) to be covered	Screw friction.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	apply knowledge of screw friction.	L3

Teaching Learning Material	Student Activity
chalk and talk	learn and solve.

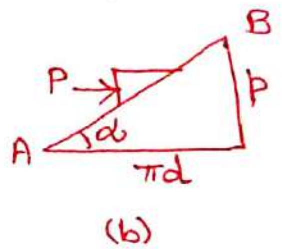
Lecture Notes



Screw friction:

A screw jack is a device used for lifting or lowering heavy loads by applying comparatively smaller force at the end of the lever. The principles used for solving problems on inclined plane are used to solve the problems of screw friction.

consider a screw shown in fig.(a).



Let d = dia of the round bar (mean dia).

p = pitch of the thread.

α = spread of thread (or angle of inclined plane, called an angle of helix).

$$\tan\alpha = \frac{p}{\pi d} = \frac{p}{2\pi r}$$ where r = mean radius of thread.

Force reqd. to move the load W up an inclined plane $\Big\}$ $P = W\tan(\phi+\alpha)$.

Let the force (or effort) applied at the end of a handle of length L $\Big\}$ $= P_1$

Taking moment about the axis of the screw,

$$P_1 \times L = P \times r = Wr\tan(\phi+\alpha).$$

$$\therefore \boxed{P_1 = \frac{Wr}{L}\tan(\phi+\alpha)}$$

P = Force reqd. at the surface of the screw up the plane i.e at the points of contact between the screw and the nut my horz. force Q, applied at the end of handle to lower the load is

$$\boxed{Q_1 = \frac{Wr}{L}\tan(\phi - \alpha)}$$

Note:

1. Q_1 is in opposite direction of P_1.
2. If angle of friction (ϕ) is larger than angle of helix (α), the screw is said to be self-locking. ie load will remain in place even after the removal of force at the end of lever.

(9) In a screw jack, the pitch of the square threaded screw is 5.5 mm and the mean diameter is 70 mm. The force exerted in turning the screw is applied at the end of a lever 210 mm long measured from the axis of the screw. If the co-eff. of friction of the screw jack is 0.07. Calculate the force reqd. at the end of the lever to (i) raise a wt. of 30 kN. (ii) lower the same weight.

Given: $p = 5.5$ mm, dia of screw $d = 70$ mm.
pitch of screw, length of lever $L = 210$ mm.
weight $W = 30$ kN, co-eff. of friction $\mu = 0.07$.
angle of friction $\phi = \tan^{-1}(\mu) = \tan^{-1}(0.07) = 4^\circ$.

(i) Force reqd. at the end of lever to raise the weight.

Force reqd. to raise the load at surface of screw.

$$P = W\tan(\phi + \alpha)$$
$$= 30\tan(4 + 1.4)$$
$$= 30\tan 5.4 = 2.835 \text{ kN}.$$

$$\tan\alpha = \frac{p}{\pi d} = \frac{5.5}{\pi \times 70}$$
$$\alpha = \tan^{-1}(0.025) = 1.4^\circ$$

Force reqd. at the end of lever;

Take moment of forces about the axis of screw,

$$P_1 \times 210 = 2.835 \times \frac{70}{2}$$
$$\boxed{P_1 = 0.4725 \text{ kN}.}$$

$$\boxed{P_1 = \frac{Wr}{l}\tan(\phi + \alpha)}$$

ii) Force required at the end of lever to lower the weight.

Force P reqd. to lower the load at the surface of the screw,

$$Q = W \tan(\phi - \alpha) = 30 \tan(4 - 1.41^\circ).$$

$$= 30 \tan 2.6^\circ.$$

$$\boxed{Q = 1.362 \text{ kN}}$$

To find force reqd. at the end of the lever, take moments of forces about the axis of the screw,

$$Q_1 \times 210 = 1.362 \times \frac{70}{2}$$

$$\boxed{Q_1 = \frac{Wr}{l} \tan(\phi - \alpha)}$$

$$\boxed{Q_1 = 0.227 \text{ kN.}}$$

Suggested Questions / Assignments / Home works / any other

study university solved question paper.

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Engineering Mechanics-Statics and Dynamics	Dr. N. Kottiswaran	Sri Balaji Publications
2.	Vector Mechanics for Engineers: Statics and Dynamics	Beer Ferdinand P, Russel Johnston Jr., David F Mazurek, Philip J Cornwell, Sanjeev Sanghi,	McGraw Higher Education
3.	Engineering Mechanics-Statics and Dynamics	Vela Murali	Oxford University Press

Any other suggested Materials

Class notes and Handouts

Lecture No. 36

UNIT IV – FRICTION

Topic(s) to be covered	Belt friction

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	analyse problems on belt friction and solve them	L3

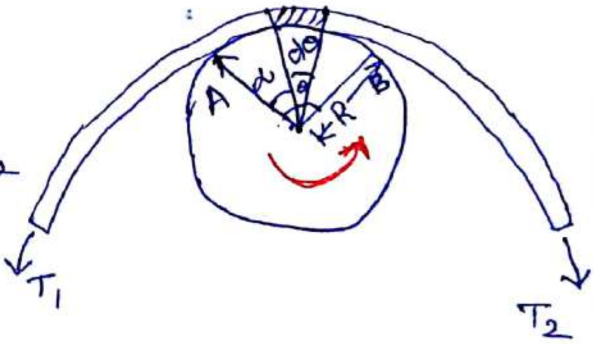
Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

Belt friction: Belt drive is a device with belt and pulley arrangement, which is used for transmitting power from one end to other end, applying brakes, lifting a load etc. In this the frictional forces developed between the belt and its contact surfaces are required to design.

Consider a pulley of radius R, with a belt is contact over an angle θ. Let the pulley rotate in anticlockwise direction.

$\therefore T_2 > T_1$



where

T_2 = tension in the tightside of the belt

T_1 = tension in the slack side of the belt.

$$\frac{T_2}{T_1} = e^{\mu\theta} \quad \text{or} \quad \boxed{T_2 = T_1 e^{\mu\theta}}$$

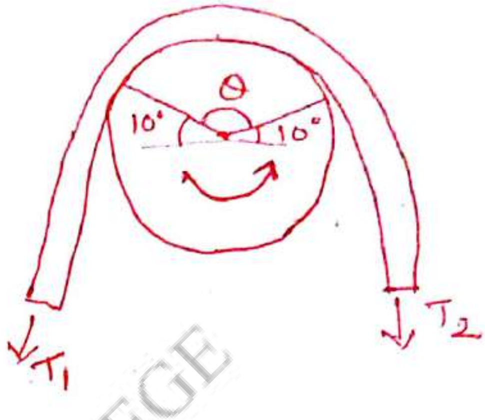
radian = degree $\times \frac{\pi}{180}$

where, angle of contact θ must be in radians.

Torque = $(T_2 - T_1) \times$ radius of the shaft.

Power transmitted = $(T_2 - T_1) \times$ velocity of belt.

(10) A rope is wound over a pulley as shown. If the tension which pulls the belt on one end is 4 KN. Determine the necessary tension on the other side of the belt to resist. Take $\mu = 0.25$.



Tension on the belt, $T_2 = 4\,KN$.

contact angle $\theta = 180° - (10+10)$

$$\theta = 160°.$$

Convert θ to radians,

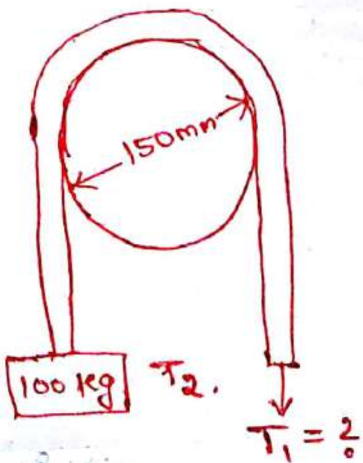
$$\theta = \left(\frac{160 \times \pi}{180}\right) \text{ radians.}$$

$$T_2 = T_1 e^{\mu\theta}.$$

$$T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{4}{e^{(0.25 \times 2.792)}}$$

$$\boxed{T_1 = 2\,KN.}$$

(11) A 100 kg mass is lifted by a rope, rolling on a cylinder of 150mm dia as shown. If the co-eff. of friction is 0.2, calculate (i) the necessary force
(ii) torque at the cylinder surface
(iii) Power transmitted.



$T_2 = 100 \times 9.81 = 981\,N$.

$\theta = 180° = 180 \times \frac{\pi}{180} = \frac{\pi}{2}$ radians.

radius $= \frac{0.15}{2}$ m. ; $T_1 = ?$

$$T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{981}{e^{(0.2 \times \pi)}} = 523.48\,N.$$

In the above problem, calculate the torque and power transmitted, if the velocity is 30m/s.

Torque at the surface of cylinder $= (T_2 - T_1) \times$ radius.

$= (981 - 523.48) \times \frac{0.15}{2}$

$= 34.314$ Nm.

Power transmitted $= (T_2 - T_1) \times$ velocity

$= (981 - 523.48) \times 30$

$= 13725.6$ Nm/s.

In the above example, if the load is lifted by applying a horizontal force as shown. calculate i) the necessary force ii) Torque at the cylinder surface iii) Power transmitted.

Given, $T_2 = 100 \times 9.81 = 981$ N.

contact angle, $\theta = 90° = \left(90 \times \frac{\pi}{180}\right) = \frac{\pi}{2}$ radians.

radius $= \left(\frac{0.15}{2}\right)$ m $\quad T_1 = ?$

Using the equation, $T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{981}{e^{(0.2 \times \pi/2)}} = 716.58$ N

Torque $= (T_2 - T_1) \times$ radius.

$= (981 - 716.58) \times \left(\frac{0.15}{2}\right)$

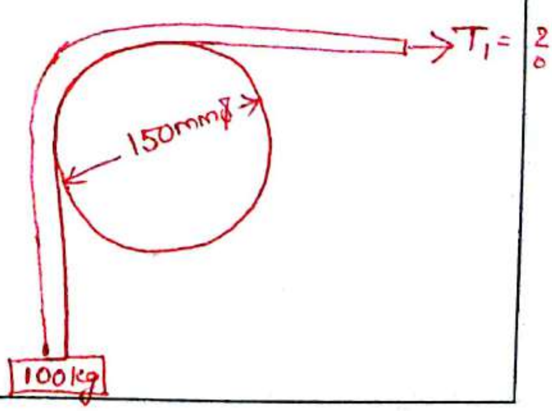
$= 19.83$ Nm.

Power transmitted

$= (T_2 - T_1) \times$ velocity

$= (981 - 716.58) \times 30$

$= 7932.6$ Nm/s.

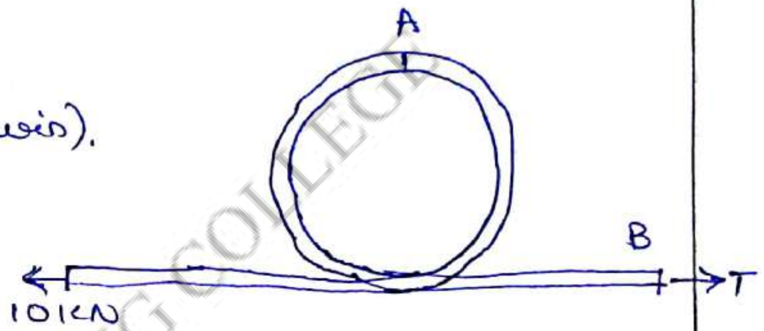


A rope is passing around a fixed pulley as shown. Determine : i) the tension in the rope at pt. A. ii) the tension in the rope at point B, to maintain equilibrium of the rope. Take $\mu = 0.25$.

i) Tension in the rope at A.

Given $T_2 = 10$ kN. , $\mu = 0.25$

T_1 = Tension which resist = tension at A.



angle of contact of belt,

$$\theta = \left(180 \times \frac{\pi}{180}\right) \quad (\because \text{half turn}).$$

$$= \pi \text{ rad.}$$

$$\therefore T = \frac{T_2}{e^{\mu\theta}} = \frac{10}{e^{0.25 \times \pi}}$$

$$= 4.56 \text{ kN.}$$

ii) Tension in the rope at B.

$$\theta = \left(360 \times \frac{\pi}{180}\right) \quad (\because \text{full turn})$$

$$= 2\pi \text{ rad.}$$

$$\therefore T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{10}{e^{(0.25 \times 2\pi)}} = 2.079 \text{ kN}$$

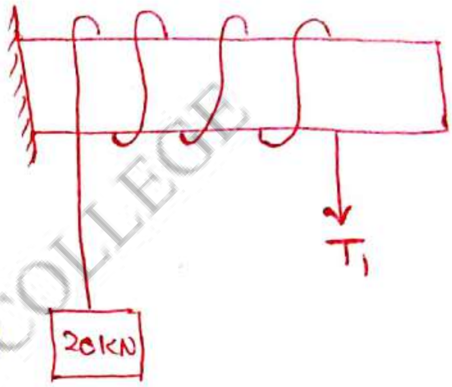
Power Transmitted = $(T_2 - T_1) \times$ velocity.

$= (981 - 716.58) \times 30$

$= 7932.6$ Nm/s.

(12) A rope is wrapped 3 times around a rod as shown. Determine the force reqd. on the free end of the rope, to support a load of 20 kN wt. The co-eff. of friction bet. the rope and rod is 0.3.

Since the rope is wrapped around the rod, the force reqd. to balance on the free end < 20 kN.



$T_2 = 20$ kN.

T_1 = unknown force required to balance on the free end.

contact angle of belt = angle of one turn × no. of turns

$= (360 \times 3) \times \frac{\pi}{180}$.

$= 6\pi$ radians.

$\therefore T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{20}{e^{(0.3 \times 6\pi)}}$

$= 0.07$ kN

$\boxed{T_1 = 70\text{ N}}$

UNIT IV (Part 1)

Dynamics of Particles

Dynamics is the part of mechanics that deals with the analysis of bodies in motion. Dynamics is divided into two parts

(i) Kinematics. chap. 16, 17, 22 → 3
(ii) Kinetics. chap. 18, 19, 20, 21, 23 → 5

Kinematics is a study of the geometry of motion. It is used to relate displacement, velocity, acceleration and time of bodies in motion, without reference to the cause of the motion.

Rectilinear – with uniform acc.
" variable ".
curvilinear – on ground.
Projectile.

Kinetics is a study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Newton's law
Work-Energy method
Impulse & momentum

Displacement, velocity and acceleration:

Displacement $= x$

Velocity $= v = \frac{dx}{dt}$

acceleration $a = \frac{dv}{dt} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{time taken}}$ (change in velocity / time taken)

Types of plane motion

1. Rectilinear motion.
2. Curvilinear motion.

The motion of a particle along a st. line is 1D.
The motion of a particle along a curved path is known as 2D.

① The motion of a particle is defined by the relation $x = t^4 - 12t^2 - 40$ where x is expressed in metres and t in seconds. Determine the position, velocity and acceleration when $t = 2s$.

The equations of motion are

$$x = t^4 - 12t^2 - 40.$$

and $v = \frac{dx}{dt} = 4t^3 - 24t$

and $a = \frac{dv}{dt} = 12t^2 - 24.$

Average velocity $= \frac{\Delta x}{\Delta t} = \frac{\text{change in position}}{\text{change in time}}$

Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

If a particle starts from a pt and returns to the same pt, average velocity is zero. But ave speed is not zero.

Position: when $t = 2s$,

$x = (2)^4 - 12(2)^2 - 40.$

$\boxed{\therefore x = -72m}$; $v = 4(2)^3 - 24(2)$ $\boxed{v = -16 m/s}$; $a = 12(2)^2 - 24$ $\boxed{a = 24 m/s^2}$

Types of rectilinear motion.

In a rectilinear motion, the particle travels along a st. line. On the st. line, the motion of particle can be with

(i) uniform acceleration.

(or) (ii) variable acceleration.

I: Rectilinear motion with uniform acceleration:

Let u = Initial velocity (m/s).

v = Final velocity (m/s).

s = Distance travelled by the particle (m).

t = time taken by the particle, to change from u to v (sec).

a = acceleration of the particle (m/s^2).

Equations of motion in a straight line:

$$\boxed{v = u + at} \quad \ldots (1)$$

due to uniform acceleration.

$$\text{average velocity} = \frac{\text{Initial velocity} + \text{Final velocity}}{2} = \frac{u+v}{2}.$$

$$\therefore \boxed{s = ut + \frac{1}{2}at^2} \quad \ldots (2)$$

$$\boxed{v^2 = u^2 + 2as} \quad \ldots (3)$$

The eqns. (1), (2) and (3) are known as equations of motion In the above equations, 'a' is linear acceleration. and if negative acceleration (or retardation) occurs, use −ve sign for 'a'.

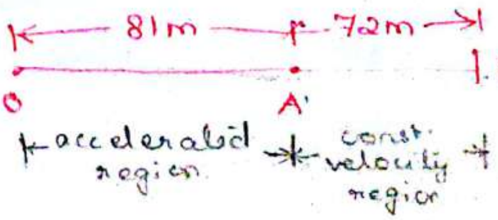
Points to remember:

1) If a body starts from rest, its initial velocity is zero (u = 0)

2) If a body comes to rest, its final velocity is zero (ie v = 0)

(2) A particle starts with an initial velocity and moves for 3 sec. with const. acceleration & travels 81m. The acceleration then ceases and during the next 3 seconds it describes 72m. Find the initial velocity and acceleration.

Let the particle starts with a velocity u and moves with a const. acceleration upto A, then velocity of the particle is constant.



Consider the motion from O to A (time 3 sec). Given: distance & time to find u, v and a

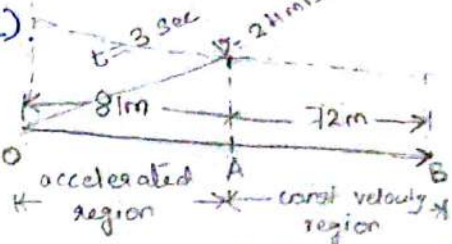
Let v be the velocity at A,

$v = u + at \quad (t = 3\,sec)$

$v = u + 3a$ ......(1)

$S = ut + \frac{1}{2}at^2 \quad (t = 3\,sec)$

$81 = 3u + 4.5a$ ......(2).



Three unknown v, u and a cannot be determined from these two equations.

Consider the motion from A to B (time 3 sec)

(final velocity at B is equal to the uniform velocity bet. A and B)

Distance travelled = uniform velocity × time taken.

$72 = v \times 3. \quad d = vt \qquad velocity = \frac{distance}{time}$

$\therefore v = \frac{72}{3} = 24\,m/s.$

Sub. v in (1),

$24 = u + 3a$ ......(3)

$24 = 30 + 3a$

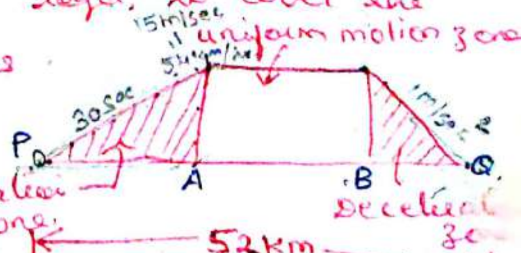
$3a = 24 - 30$

$a = \frac{-6}{3} = 2\,m/s^2$

Solving eqns. (2) and (3),

Initial velocity $u = 30\,m/s$. and retardation $a = -2\,m/s^2$.

(3) Two stations P and Q are 52 km apart. A train starts from rest at the station P and accelerates uniformly to attain a speed of 54 km/hr in 30 seconds. This speed is maintained until the brakes are applied. The train comes to rest at the station Q with uniform retardation of $1\,m/sec^2$. Determine the total time reqd. to cover the distance between these two stations.



Let the train starts from P and attains a velocity of 54 km/hr (= 15 m/sec) at A.

$= \frac{54 \times 1000}{60 \times 60} = 15\,m/sec$

Let the brakes are applied at B and comes to rest at Q.

Consider the motion from P to A.

$u = 0; \quad v = 15\,m/sec; \quad t = 30\,sec.$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

using $v = u + at$.

$15 = 0 + (a \times 30) \quad \therefore \boxed{a = 0.5\,m/sec^2}$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$\therefore$ Distance $\underline{PA} = s = ut + \frac{1}{2}at^2$

$= 0 + \left(\frac{1}{2} \times 0.5 \times 30^2\right)$

$= 225 m.$

consider the motion from B to Q.

$u = 15 m/sec$; $v = 0$; $a = -1 m/sec^2$, $t = ?$

using $v = u + at$

$0 = 15 - (1 \times t)$

$\therefore \boxed{t = 15 sec.}$

$\therefore$ Distance $\underline{BQ} = s = ut + \frac{1}{2}at^2$

$= (15 \times 15) - \left(\frac{1}{2} \times 1 \times 15^2\right)$

$= 112.5 m.$

consider the motion from A to B.

velocity = $\frac{distance}{time}$

Distance = 54000 − 225 − 112.5.

$v = \frac{x}{t}$

$t = \frac{x}{v}$

AB = 53662.5 m.

$\therefore$ time taken $= \frac{53662.5}{15 \text{ velocity}} = \cancel{3577.5}$ 3444.17 sec

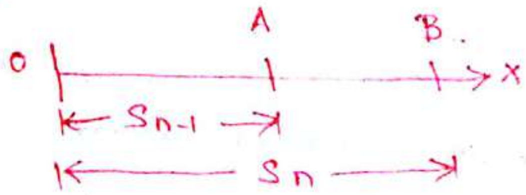
$\therefore$ Total time required to cover the distance between these two stations $= 30 + \cancel{3577.5}$ 3444.17 $+ 15$

$= \cancel{3622.5}$ sec. or 60 min 22.5 sec.

3489.17 sec. or 58.15 min.

Distance travelled in n^{th} second:

~ is determined by subtracting the distance travelled by the particle in (n−1) sec. from the distance travelled by the same particle in n seconds



Distance travelled in n^{th} second (AB) = The difference between S_n and S_{n-1}, we know, $S = ut + \frac{1}{2}at^2$.

$$\boxed{S^{nth} = u + \frac{a}{2}(2n-1)}$$

o 9 10

Lecture No.

UNIT V – DYNAMICS OF PARTICLES

Topic(s) to be covered	

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

Motion of a particle under Gravity: is a special case of rectilinear motion under const. acceleration, known as the acceleration due to gravity, denoted as 'g' equal to $9.81\ m/s^2$

* In downward motion of particles, g is positive
* In upward motion of particles, g is negative.

Here 'a' is replaced by 'g' and the distance 's' is replaced by the height 'h'.

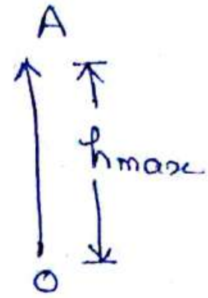
S.No	Rectilinear Motion		
	Horizontal motion	Vertical downward motion	Vertical upward motion
1	$v = u + at$	$v = u + gt$	$v = u - gt$
2	$s = ut + \frac{1}{2}at^2$	$h = ut + \frac{1}{2}gt^2$	$h = ut - \frac{1}{2}gt^2$
3	$v^2 = u^2 + 2as$	$v^2 = u^2 + 2gh$	$v^2 = u^2 - 2gh$.
4	$S_n = u + \frac{a}{2}(2n-1)$	$h_n = u + \frac{g}{2}(2n-1)$	$h_n = u - \frac{g}{2}(2n-1)$

Points to Remember:

1. When a body starts moving vertically downwards, its initial velocity $u=0$.
2. When a body is projected vertically upwards, at the highest point, its final velocity $v=0$.

Important Results: upward motion:

① Max. height attained by upward particle:



$v^2 = u^2 - 2gh$. @ $h = h_{max}$; $v = 0$.

$0 = u^2 - 2g h_{max}$

$$\boxed{h_{max} = \frac{u^2}{2g}} \quad \text{------ ①}$$

② Time taken by the particle to reach max height.

$v = u - gt$ @ $h = h_{max}$, $v = 0$.

$0 = u - gt \Rightarrow u = gt$

$$\boxed{t = \frac{u}{g}} \quad \text{------ ②}$$

Time up = time down.

∴ total time taken by the particle to return to the surface (T) $\Big\}$ = 2 × time up.

$$\boxed{T = \frac{2u}{g}} \quad \text{------ ③}$$

From ① and ②, the min. initial velocity of the particle reqd. to reach the max. ht,

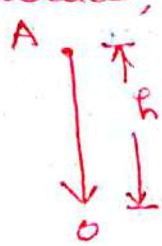
$u = \sqrt{2g \times h_{max}}$. [if h_{max} is known]

or

$u = t \times g$ [if time is known]

Important result – Downward motion:

Striking velocity of the particle, moving downwards, from a position of rest



Here initial velocity $u = 0$.

using the relation, $v^2 = u^2 + 2gh$

$= 0 + 2gh$

$$\boxed{v = \sqrt{2gh}.}$$

A stone is thrown vertically upwards. It reaches the max ht. of 12 m. Determine,

(i) the velocity with which the stone was thrown.

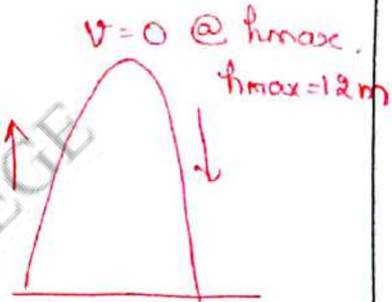
(ii) the time taken to reach max. height.

(iii) Total time taken by the stone, to return to the ground surface, after projected upwards.

(i) Velocity with which the stone was thrown. (u):

At max. ht, velocity $v = 0$.

i.e @ $h_{max} = 12\,m$, $v = 0$.



$v = u - gt$

$h = ut - \frac{1}{2} g t^2$

$v^2 = u^2 - 2gh$.

using the equation, $v^2 = u^2 - 2gh$.

$$0 = u^2 - 2gh.$$

$$u = \sqrt{2g\,h_{max}}$$

$$= \sqrt{2 \times 9.81 \times 12}$$

$$\boxed{u = 15.34\ m/s.}$$

(ii) Time taken to reach max. height. (t):

At max. height, velocity $v = 0$.

using the equation, $v = u - gt$

$$0 = 15.34 - (9.81 \times t)$$

$$t = \frac{15.34}{9.81}$$

$$\boxed{t = 1.56\ sec.}$$

(iii) Total time taken (T): is equal to twice the time taken by the stone to reach the max. height

$T = 2t = 2 \times 1.56$ sec.

$$\boxed{T = 3.12\ sec.}$$

Relative velocity - Basic concept.

Two motors A and B are moving in same direction with velocities 'u' m/s and 'v' m/s.

$A \xrightarrow{u}$

$B \xrightarrow{v}$

Relative velocity of B w.r.t. A. is $(v-u)$

i.e. $V_{B/A} = V_B - V_A = (v-u)$ m/s.

Relative velocity of A w.r.t. B is $(u-B)$

$V_{A/B} = V_A - V_B = (u-v)$ m/s.

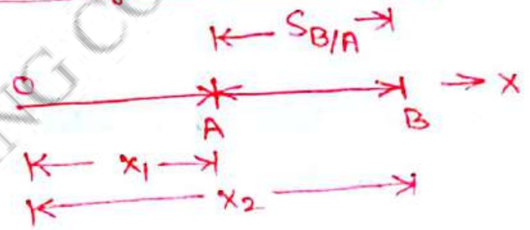
we will see relative ~~velocity~~ motion of a particle in the foll two cases.

(i) Relative velocity of two particle moving in a st. line.

(ii) Relative velocity of two particles in a plane.

(i) Relative velocity of two particles moving in a straight line.

$\therefore V_{B/A} = V_B - V_A$.



① a) cars moving in same direction.

A $\xrightarrow{V_A = 30\,m/s}$

B $\xrightarrow{V_B = 20\,m/s}$

velocity of car A relative to car B,

$V_{A/B} = V_A - V_B = 30-20 = 10$ m/s $(\rightarrow)$ +ve velocity

velocity of car B relative to car A,

$V_{B/A} = V_B - V_A = 20-30 = -10$ m/s. $(\leftarrow)$ -ve velocity.

b) Cars moving in opp. direction.

A $\xrightarrow{V_A = 30\,m/s}$

$V_B = 20\,m/s \xleftarrow{}$ B.

$V_A = 30$ m/s

$V_B = -20$ m/s. ($\because$ opp. direction).

velocity of car A relative to car B,

$V_{A/B} = V_A - V_B = 30-(-20) = 50$ m/s $(\rightarrow)$

velocity of car B relative to car A,

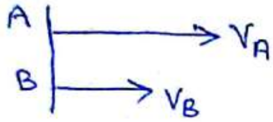
$V_{B/A} = V_B - V_A. = -20-30 = -50$ m/s. or 50 m/s $(\leftarrow)$.

Finding the relative velocity of a particle from relative velocity diagram.

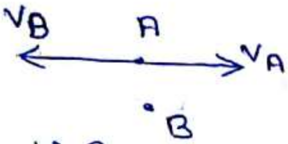
For st. line – sign convention is easy.

If the particles are moving on inclined lines, or when the particle moves in a plane, sign convention is difficult. Hence relative velocity diagram is drawn and from which relative velocity is found out.

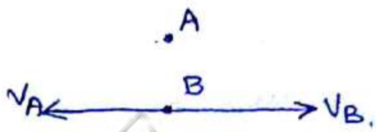
Relative velocity diagram:



a) Actual velocity diagram.



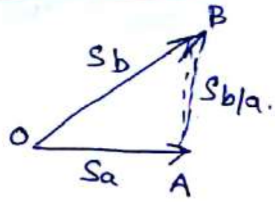
b) Relative velocity diagram to find $V_{A/B}$.



c) Relative velocity diagram to find $V_{B/A}$.

Relative velocity of two particles moving in a plane:

Consider two particles A and B moving in the same plane



$$S_{B/A} = S_B - S_A. \quad \text{(relative displacement)}$$

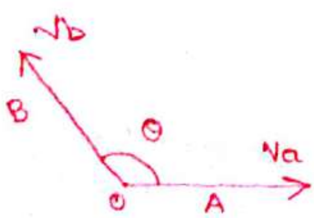
relative velocity

$$V_{B/A} = V_B - V_A.$$

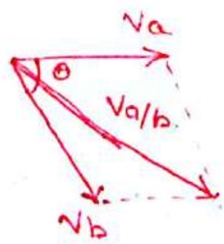
relative acceleration

$$a_{B/A} = a_B - a_A.$$

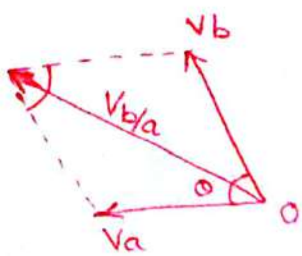
Relative velocity diagram:



a) Actual velocity diagram



b) Relative velocity diagram to find $V_{A/B}$.

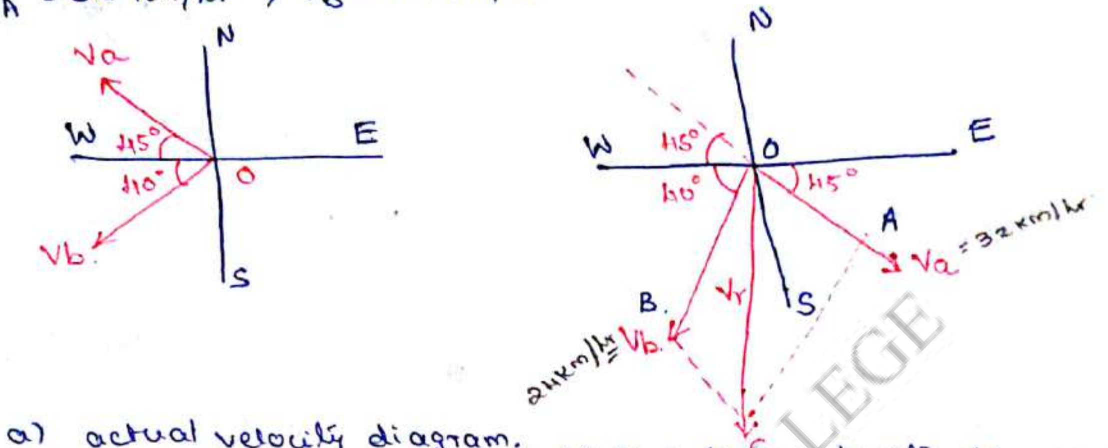


c) Relative velocity diagram to find $V_{B/A}$.

Note: To find relative of A w.r.t. B, i.e. $V_{A/B}$, reverse the direction of V_B at O. and vice versa.

(2) Two ships leave a port at the same time. First ship steams north-west at 32 km/hr and the second 40° south of west at 24 km/hr. What is the speed of the second ship relative to the first ship in km/hr. Also find the time at which the ships will be 160 km apart.

$V_A = 32$ Km/hr , $V_B = 24$ Km/hr



a) actual velocity diagram. b) Relative velocity diagram.

to find relative velocity of B w.r.t A, reverse the direction of A and apply parallelogram law of forces on OACB.

$$V_{B/A} = \sqrt{V_a^2 + V_b^2 + 2V_aV_b\cos\angle AOB}\,.$$

$V_a = 32$ km/hr. , $V_b = 24$ km/hr.

$$\cos\angle AOB = \cos(180-45-40) = \cos 95^\circ.$$

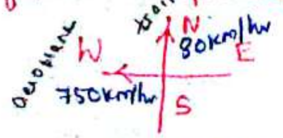
$$V_{B/A} = \sqrt{32^2 + 24^2 + (2\times32\times24\times\cos95^\circ)} = 38.29\text{ km/h}$$

The distance between the two ships at any time 't' is given by $S_{b/a} = V_{b/a} \times t$

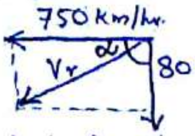
$$\therefore t = \frac{S_{b/a}}{V_{b/a}} = \frac{160}{38.29} = 4.18\text{hrs.}$$

$\therefore$ ship will be at a distance of 160 km after 4.18 hrs.

(1) An aeroplane flying at a 750 km/hr, towards west passes over a train which is travelling at 80 km/hr, towards north. Calculate velocity of the aeroplane relative to velocity of the train.



actual velocity diagram



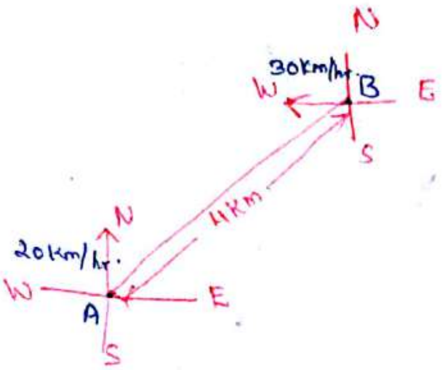
Relative velocity diagram.

Let V_r be the relative velocity of aeroplane w.r.t. train

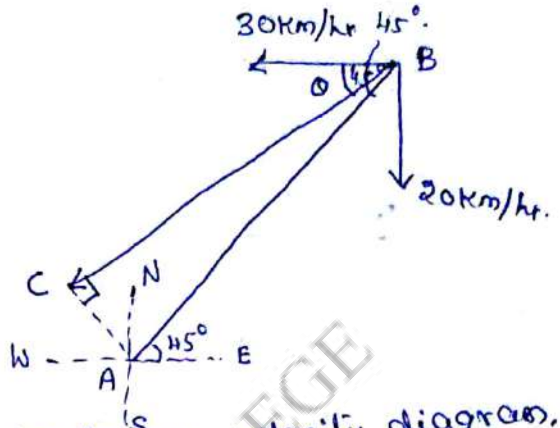
$$V_r = \sqrt{80^2 + 750^2} = 754.25\text{ km}$$

$$\alpha = \tan^{-1}\left(\frac{80}{750}\right) = 6^\circ \text{ South of West}$$

A ship steaming towards North at 20 km/hr. sights another ship, steaming at 30 km/hr, towards west at a distance of 4 km in North-Eastern direction. Determine the shortest distance between the ships.



(a) actual velocity diagram.



(b) relative velocity diagram.

Relative velocity of B w.r.t. A,

$$V_{B/A} = \sqrt{20^2 + 30^2} = 36.05 \text{ km/hr.}$$

Let θ be the direction of $V_{B/A}$ with horizontal i.e. western direction,

$$\therefore \tan\theta = \frac{20}{30} \Rightarrow \boxed{33.69^\circ = \theta}$$

The shortest distance bet. the ships is AC.

In rt. angled triangle ABC, rt. angled at C,

$$\angle CBA = 45 - \theta = 45 - 33.69 = 11.31^\circ.$$

$\therefore$ shortest distance $AC = AB \sin \angle CBA = 4 \sin 11.31 = 0.784 \text{ km}.$

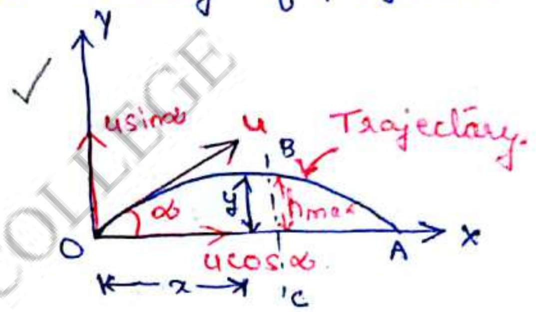
Projectile Motion.

Definitions:

1. Projectile – A particle projected in space at an angle to the horizontal plane is called a projectile.

2. Angle of projection – The angle to the horizontal, at which the projectile is projected is called angle of projection. It is denoted by 'α'.

3. Velocity of projectile – The velocity with which the projectile is thrown into space is called the velocity of projectile denoted by 'u' measured in m/s.



component of velocity along ox axis $= u\cos\alpha$ and

component of velocity along oy axis $= u\sin\alpha$.

4. Trajectory – The path described by the projectile is called Trajectory.

5. Time of Flight – It is the total time taken by the projectile from the instant of projection upto the projectile hits the plane again.

6. Range – It is the distance along the plane between the pt. of projection and the pt. at which the projectile hits the plane at the end of its journey.

Path of projectile:

$$\boxed{\therefore y = \tan\alpha\, x - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}}$$

is derived from $\boxed{h = ut - \frac{1}{2}gt^2}$ where $u = u\sin\alpha$, $h = y$.

This equ. is of the form $y = Ax + Bx^2$. represents a parabola and hence the path traversed by the projectile is a parabola. The above equ. is the equ. of trajectory.

horz. distance travelled by projectile $\boxed{x = u\cos\alpha t}$ vertical distance travelled by projectile $\boxed{y = u\sin\alpha t - \frac{1}{2}gt^2}$

Standard results:

From the equation of trajectory, it is clear that the two variables of projectile are initial velocity (u) and angle of projection (α). Now the foll. std results of projectile motion are arrived in terms of u and α.

(i) Time of flight:

OB - upward motion of the projectile.

BA - downward " " " ".

Let t be the time taken by the projectile to reach the max. ht. B.

$$\boxed{t = \frac{u \sin\alpha}{g}}$$

total time taken (or) Time of flight (T) $= 2t = 2 \times \dfrac{u\sin\alpha}{g}$

[" " from O to A].

$$\therefore \boxed{T = \frac{2u\sin\alpha}{g}}$$

(ii) Max. height attained: pt. reached by the projectile $= h_{max}$.

pt. B is the highest pt. reached by the projectile

$$\boxed{h_{max} = \frac{u^2 \sin^2\alpha}{2g}}$$

(iii) Horizontal range: (distance OA).

$$\boxed{\text{Range } R = \frac{u^2 \sin 2\alpha}{g}}$$

max. horizontal range; for the given value of u, the horz. range R will be max. when $\sin 2\alpha$ has max. value which is equal to unity.

$$\therefore \sin 2\alpha = 1.$$

$$2\alpha = \sin^{-1}(1) = 90.$$

$$\alpha = 45^\circ.$$

For the given value of u, the horz. range will be max. if its angle of projection α is 45°.

1) A body is projected at an angle such that its horizontal range is 3 times the max height. Find the angle of projection. (Anna Nov/Dec 2002)

Let α be the angle of projection.

It is given $R = 3h_{max}$.

$$\frac{u^2 \sin 2\alpha}{g} = 3\left(\frac{u^2 \sin^2\alpha}{2g}\right)$$

$$\sin 2\alpha = 1.5 \sin^2\alpha.$$

$$2\sin\alpha \cos\alpha = 1.5 \sin^2\alpha$$

$$2\cos\alpha = 1.5 \sin\alpha.$$

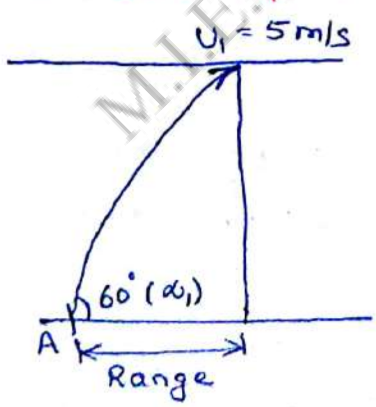
$$\frac{\sin\alpha}{\cos\alpha} = \frac{2}{1.5}.$$

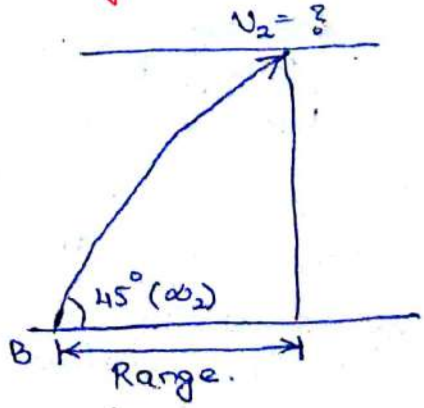
$$\tan\alpha = 1.333.$$

$$\therefore \alpha = \tan^{-1}(1.333) = 53.13^\circ.$$

2) A projectile is thrown with a velocity of 5 m/sec at an elevation of 60° to the horizontal. Find the velocity of another projectile thrown at an elevation of 45° which will have (i) equal horizontal range (ii) equal max. height and (iii) equal time of flight with the first.

Solution. (i) with equal horizontal range.





We know horizontal range $R = \frac{u^2 \sin 2\alpha}{g}$.

For first stone, $u_1 = 5$ m/s and $\alpha_1 = 60°$

$$\therefore R_1 = \frac{5^2 \sin(2\times 60)}{9.81}$$

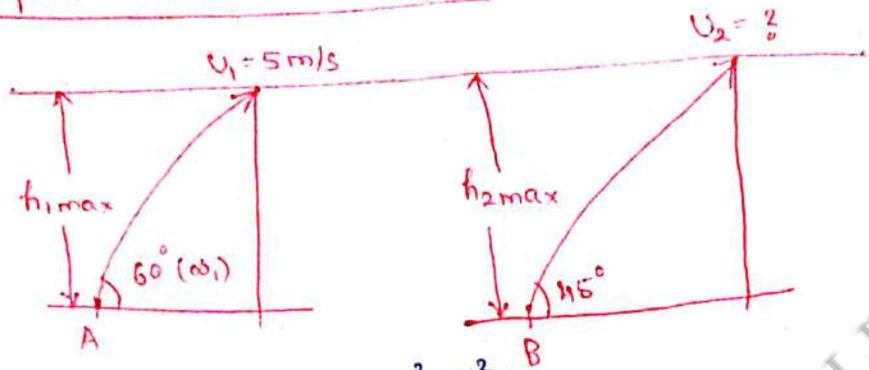
Now, $R_1 = R_2$

$$\therefore \frac{5^2 \sin 120^\circ}{9.81} = \frac{u_2^2 \sin 2\alpha_2}{g} \quad (\text{but } \alpha_2 = 45^\circ)$$

$$\text{or} \quad \frac{5^2 \sin 120}{9.81} = \frac{u_2^2 \sin(2 \times 45^\circ)}{9.81}$$

$$u_2 = 4.653 \text{ m/s.}$$

(ii) with Equal maximum height :



we know, $h_{max} = \dfrac{u^2 \sin^2 \alpha}{2g}$.

Now, $h_1 max = h_2 max$.

$$\frac{u_1^2 \sin^2 \alpha_1}{2g} = \frac{u_2^2 \sin^2 \alpha_2}{2g}$$

Substituting the values,

$$\frac{5^2 \sin^2 60^\circ}{2 \times 9.81} = \frac{u_2^2 \sin^2 45^\circ}{2 \times 9.81}$$

$$\boxed{u_2 = 6.124 \text{ m/sec}}$$

(iii) with equal time of fight :

we know, time of flight $T = \dfrac{2u\sin\alpha}{g}$

Now, $T_1 = T_2$.

$$\frac{2u_1 \sin\alpha_1}{g} = \frac{2u_2 \sin\alpha_2}{g}.$$

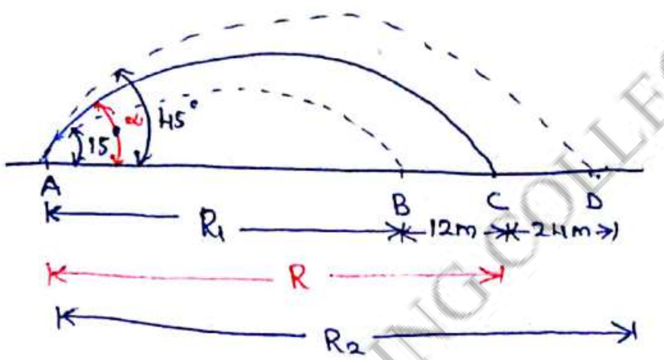
Sub. the values,

$$\frac{2 \times 5 \times \sin 60^\circ}{9.81} = \frac{2 \times u_2 \times \sin 45^\circ}{9.81}.$$

$$\boxed{u_2 = 6.123 \text{ m/s.}}$$

③ A projectile is aimed at a mark on the horizontal plane thro' the pt. of projection and falls 12m short when the angle of projection is 15°. When it is tried again it overshoots the mark by 24m when the angle of projection is 45°. Find the correct angle of projection to hit the mark. Velocity of projection is const. in all the cases.

Let R be the range of mark and α be the angle of projection to hit the mark.



Case (i):

Range $R_1 = (R-12)m.$; $\alpha_1 = 15°$

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$$(R-12) = \frac{u^2 \sin(2\times 15)}{g}$$

$$(R-12) = \frac{u^2 \sin 30}{g}$$

$$(R-12) = \frac{0.5u^2}{g} \quad \ldots (1)$$

Case (ii):

Range $R_2 = (R+24)m$, $\alpha = 45°$

Ily

$$(R+24) = \frac{u^2 \sin(2\times 45)}{g} = \frac{u^2 \sin 90}{g}$$

(or) $$(R+24) = \frac{u^2}{g} \quad \ldots (2)$$

Dividing equ. (1) by (2);-

$$\frac{(R-12)}{(R+24)} = \frac{(0.5u^2/g)}{(u^2/g)}$$

$$\frac{R-12}{R+24} = 0.5 \Rightarrow R-12 = 0.5(R+24). \Rightarrow R = 48m.$$

Sub. R in (1),

$$(R-12) = \frac{0.5u^2}{g} \Rightarrow (48-12) = \frac{0.5u^2}{9.8} \Rightarrow u = 26.58 \text{ m/s}.$$

Now, Knowing the range of mark and velocity of projection, the angle of projection α can be determined using the relation,

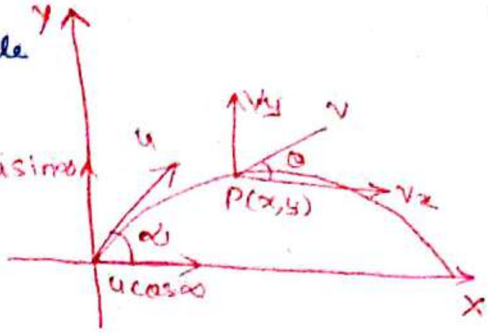
$$R = \frac{u^2 \sin 2\alpha}{g} \Rightarrow 48 = \frac{(26.58)^2 \sin 2\alpha}{9.81} \Rightarrow \sin 2\alpha = 0.666.$$

$$2\alpha = \sin^{-1}(0.666) = 41.76° \quad \therefore \alpha = \frac{41.76}{2} = 20.8°.$$

∴ The correct angle of projection to hit the mark is 20.8°

① Position and velocity of the projectile after a known time

Let $P(x,y)$ be the position of the projectile after a given time 't'.



V_x = horz. velocity of projection 'u = const.

$\therefore \quad V_x = u\cos\alpha$

Initial vertical velocity $= u\sin\alpha$

using the equ. $V = u - gt$

$V_y = u\sin\alpha - gt$

$\therefore$ velocity at P, $V = \sqrt{(V_x)^2 + (V_y)^2}$

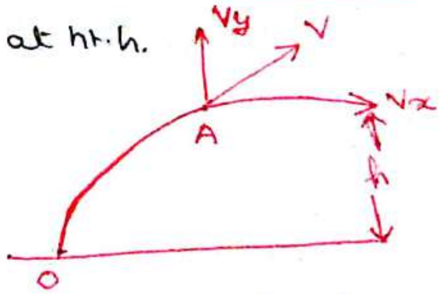
distance = vel. x t

Direction of velocity with horizontal $\Big\}$ $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$.

Position of P : $\boxed{\begin{array}{l} x = u\cos\alpha t \text{ and} \\ y = u\sin\alpha t - \frac{1}{2}gt^2 \end{array}}$

② Velocity and direction of the projectile after a known ht.

Let v be the velocity of projectile at ht. h. (at pt. A).



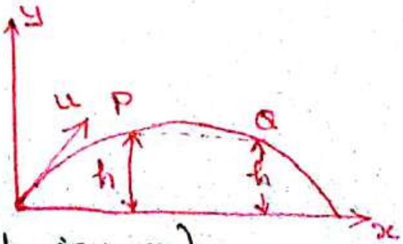
Horz. component of velocity at A $\Big\}$ $V_x = u\cos\alpha$

vertical component of velocity at A $\Big\}$ $V_y = \sqrt{(u\sin\alpha)^2 - 2gh}$ $\qquad v^2 = u^2 - 2gh$.

$\therefore$ Resultant velocity $V = \sqrt{(V_x)^2 + (V_y)^2}$ and $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$.

③ Time taken by the projectile at a known height:

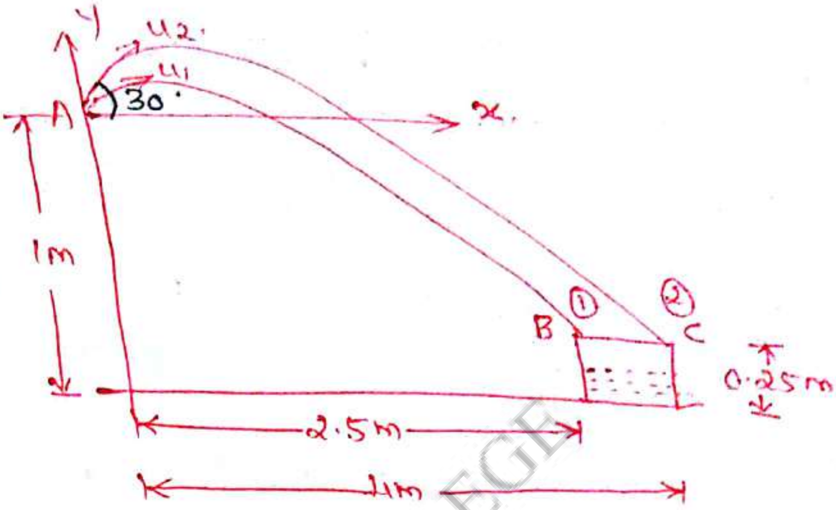
Let 't' be the time taken by the projectile at the given ht. h.



$$\boxed{gt^2 - 2u\sin\alpha t + 2h = 0}$$

solving the quadratic equ. we get two values for t say t_1 and t_2. (upward journey and downward journey).

(4) A boy throws two stones at an angle of 30° from pt A as shown. Determine the time between throws so that both stones strike the edges of the tank B and C at the same instant. With what speed must he throw each stone.



Let the boy throws two stones (1) and (2) to strike tank edges B and C respectively.

Projectile (1):

u_1 = velocity ; $\alpha = 30°$; Range = 2.5m.

$\therefore$ co-ordinates of B = (2.5, −0.75)

Sub. the above values in the equ. of trajectory,

$$\boxed{y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}}$$

$$-0.75 = 2.5\tan 30° - \frac{9.81 \times 2.5^2}{2 \times u_1^2 \cos^2 30°}$$

$$-0.75 = 1.443 - \frac{40.875}{u_1^2}$$

$$u_1 = \sqrt{\frac{40.875}{(1.443 + 0.75)}} \Rightarrow \boxed{u_1 = 4.317 \text{ m/s.}}$$

Projectile (2):

velocity = u_2, $\alpha = 30°$, Range = 4m., co-ordinate C = (4, −0.75)

sub. in equ. of trajectory,

$$-0.75 = 4\tan 30° - \frac{9.81 \times 4^2}{2 \times u_2^2 \times \cos^2 30°}$$

$$\therefore -0.75 = 2.309 - \frac{104.64}{u_2^2}$$

$$\boxed{u_2 = 5.849 \text{ m/s.}}$$

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Time difference between throws: (t_1 and t_2) $gt^2 - 2u\sin\alpha t + 2h = 0$

or use: $9.81t^2 - (2\times 4.317)\sin 30° t + 2\times(-0.75) = 0$

We know, Range $R = u\cos\alpha \times t$.

$R_1 = u_1\cos\alpha t_1 \Rightarrow t_1 = \frac{R_1}{u_1\cos\alpha} = \frac{2.5}{4.317 \times \cos 30°}$

$9.81t^2 - 4.317t - 1.5 = 0$

$\boxed{t_1 = 0.668 \text{ sec.}}$

$t = 0.669 \text{ sec.}$

IIIly $R_2 = u_2\cos\alpha t_2 \Rightarrow t_2 = \frac{R_2}{u_2\cos\alpha} = \frac{4}{5.849 \times \cos 30°}$

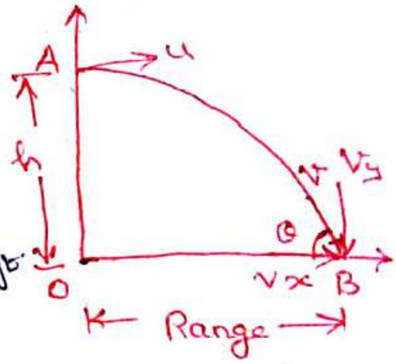
$\boxed{t_2 = 0.789 \text{ sec}}$

Time difference between two throws $= (t_2 - t_1)$

$= 0.789 - 0.668$

$= 0.121 \text{ sec}$

case II: Motion of particle thrown horizontally from a known ht:



at A: u = horz. velocity with which particle is thrown.

here vertical velocity $= 0 = v_y$

at B: horizontal velocity $v_x = u$ (horz. velocity at A)

vertical velocity $\boxed{v_y = gt}$ $v_y = u\sin\alpha + gt$, $\sin\alpha = 0$

Resultant velocity at B, $v = \sqrt{v_x^2 + v_y^2}$

$\boxed{v = \sqrt{u^2 + (gt)^2}}$

Direction of velocity with horizontal θ $\Big\}$ $= \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{gt}{u}\right)$.

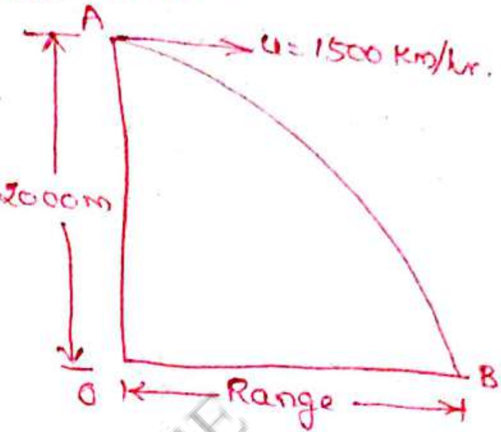
Range: Range = horz. velocity × time taken.

$\boxed{R = u \times t.}$

Vertical distance travelled by the particle is $h = ut + \frac{1}{2}gt^2$.

here $u = u\sin\theta = 0$ $\therefore$ $\boxed{h = \frac{gt^2}{2}.}$

(5) A bomb is released from an aeroplane, flying at a speed of 1500km/hr on a st. line, 2000m above the ground. Determine the time reqd. for the bomb to reach the ground and the horz. distance travelled by the bomb.



Velocity of aeroplane = velocity of bomb.

$u = 15\,Km/hr = \frac{1500 \times 1000}{3600} = 416.67\,m/s$

Ht of aeroplane $h = 2000m$.

Time reqd. to reach the ground.

using $h = ut + \frac{1}{2}gt^2$, here $h = 2000m$.

$\therefore\ 2000 = 0 + \left[\frac{1}{2} \times 9.81 \times t^2\right]$

$$\boxed{t = 20.2\,sec.}$$

Horizontal distance travelled:

Range R = horz. velocity × time taken = ut

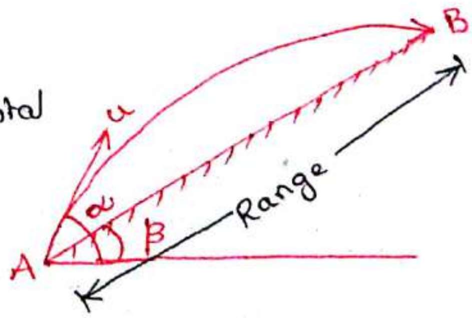
$= 416.67 \times 20.2 = 8416.7m.$

case III: Projection up an inclined plane.

if u = velocity of projection.

α = angle of projection with horizontal

β = angle of inclined plane with ".



Time of flight $\boxed{T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}.}$

Range $\boxed{R = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}.}$

Maximum Range: $\boxed{R_{max} = \frac{u^2}{g(1+\sin\beta)}}$

[condition for max. Range is $(2\alpha-\beta) = 90° = \frac{\pi}{2}$.]

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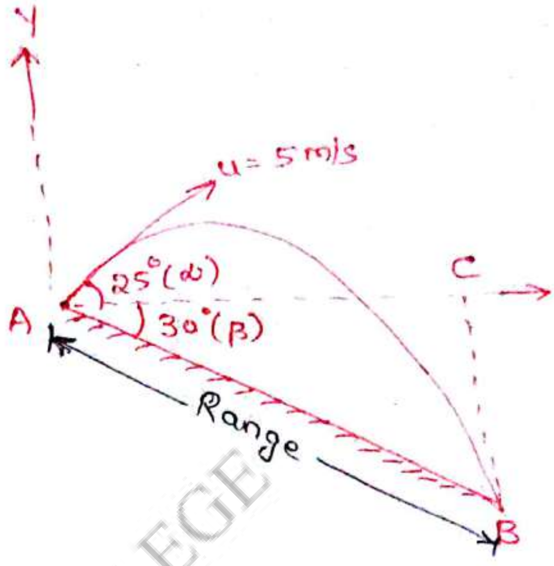
(7) A ball is projected from A with velocity 5 m/s at an angle of 25° as shown. Determine the horizontal and vertical distances of B, which the ball hits the plane, which is 30° below the horizontal.



Given: $u = 5\,m/s$; $\alpha = 25°$ and $\beta = 30°$

Range of projectile,

$$R = \frac{2u^2 \cos\alpha \sin(\alpha+\beta)}{g \cos^2\beta}.$$

(down the plane case)

$$R = \frac{2 \times 5^2 \times \cos 25 \times \sin(25+30)}{9.81 \times \cos^2 30}$$

$$\boxed{R = 5.045\,m.}$$

Horizontal distance travelled:

In rt. 2nd Δ^{le} ABC,

horz. distance travelled (AC) = AB cos θ

$= 5.045 \times \cos 30° = 4.369\,m.$

Vertical distance travelled:

Vertical distance travelled BC = AB sin 30°

$= 5.045 \times \sin 30°$

$= 2.522\,m.$

Lecture No. 37

UNIT V – DYNAMICS OF PARTICLES

Topic(s) to be covered	Introduction to Dynamics of particles.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	Understand the concept behind Dynamics of particles – kinetics and kinematics	L2

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

UNIT V – Dynamics of Particles.

Syllabus.

Kinematics – Rectilinear motion and curvilinear motion of particles. Kinetics – Newton's second Law of motion – Equations of motions, Dynamic equilibrium, Energy and momentum Methods – Work of a force, kinetic energy of a particle, Principle of work and energy, Principle of Impulse and momentum, Impact of bodies.

Dynamics is the part of mechanics that deals with the analysis of bodies in motion. Dynamics is divided into two parts. i) Kinematics
ii) Kinetics.

Kinematics is a study of the geometry of motion. It is used to relate displacement, velocity, acceleration and time of bodies in motion, without reference to the cause of the motion.
- Rectilinear
- Curvilinear.

Kinetics is a study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

D'Alembert's Principle: is an application of Newton's 2nd law of motion. For static equilibrium of forces in plane, we have $\Sigma H = 0$, $\Sigma V = 0$ & $\Sigma M = 0$.

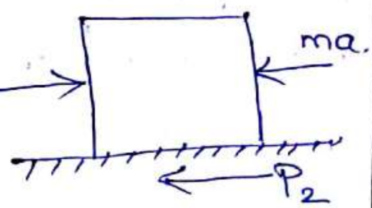
For dynamic equilibrium of body, use D'Alembert's principle.

$P = ma$ or $\boxed{P - ma = 0}$ is called as dynamic equation of equilibrium. 'ma' is called an imaginary force or inertia force always applied in opp. direction of motion.

Instead of a single force, if a system of forces are acting on a particle, then

$$\Sigma F = ma \quad \text{or} \quad \boxed{\Sigma F - ma = 0}$$

When a body is subjected to two forces P_1 and P_2, then the resultant force is $P = P_1 - P_2$ (assuming $P_1 > P_2$)



Applying $\Sigma F = 0$ ($\rightarrow$ +ve)

$$P - ma = 0$$

$$P_1 - P_2 - ma = 0 \Rightarrow \boxed{a = \frac{(P_1 - P_2)}{m}}$$

D'Alembert's Principle states that "the system of forces acting on a body in motion is in dynamic equilibrium, with the inertia force of the body.".

Laws of motion / Newton's laws of motion / Principles of motion / Principles of Dynamics:

First Law: Every body continues to be in its state of rest or in uniform motion in a st. line unless and until it is acted upon by some external force to change that state.

Second Law: The rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force applied.

Third Law: To every action, there is always an equal and opposite reaction.

Newton's Second Law of motion

The change in momentum = Final momentum − Initial momentum

$$= mv - mu = m(v-u).$$

$\therefore$ rate of change of momentum $= \dfrac{\text{change of momentum}}{\text{Time taken}}$

$$= \frac{m(v-u)}{t} = m.a. \quad \left[\because \frac{v-u}{t} = a\right].$$

As per Newton's second law,
external force P is directly proportional to the rate of change of momentum i.e. $P \propto ma \Rightarrow P = kma$.
where k & m are constant.

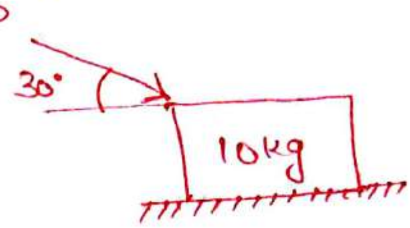
Method.	Suitability.
D'Alembert's Principle	Kinetic problems involving force and acceleration. $P = ma$.
Work - Energy method.	Kinetic problems involving force, velocity and displacement. $F \times S = \frac{1}{2}mv^2$
Impulse - Momentum method.	Kinetic problems involving force time and velocity $Ft = m(v-u)$

Q ① A block of mass 10kg, rests on a horz. plane as shown. Find the magnitude of the force P, reqd. to move the block at an acceleration of $2\,m/s^2$ towards right. Take the co-eff. of friction between the block and the plane as 0.25.

Given: $m = 10kg \Rightarrow W = mg = 10 \times 9.81 = 98.1N$.

$a = 2m/s^2$

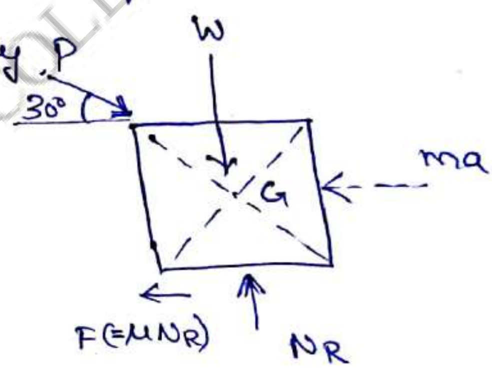
$\mu = 0.25$.



The FBD is shown in fig.

Due to the applied load, the body will move towards right. Hence inertia force will act in opp direction, passing thro' the C.G. of the body.

Now, with the body is in static equilibrium.



Applying $\Sigma V = 0$ (↑ +ve)

$$N_R - W - P\sin\theta = 0$$

$$N_R - 98.1 - P\sin 30^\circ = 0.$$

$$N_R = 98.1 + 0.5P \quad \text{------} ①$$

Applying $\Sigma H = 0$ (→ +ve).

$$P\cos\theta - F - ma = 0 \quad \text{-----} ②$$

Frictional force $F = \mu N_R$

$$= 0.25\,(98.1 + 0.5P).$$

$$= 24.52 + 0.125P.$$

Sub. F in equ. ②,

$$P\cos 30^\circ - (24.52 + 0.152P) - (10 \times 2) = 0.$$

$$0.741P = 44.52$$

$$P = 60N.$$

∴ Force reqd. to move the block is 60N.

Topic(s) to be covered	Motion of a lift

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	analyse and solve problems associated with motion of a lift	L3

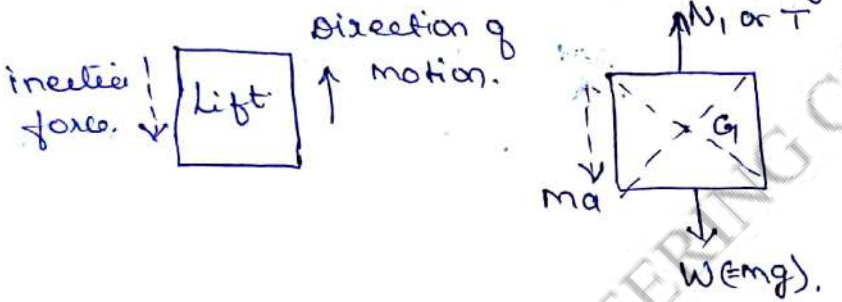
Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

Motion of a Lift:

case I : Lift is moving upwards:

when lift is moving upwards, net force is acting upwards and hence inertia force is acting downwards.



Applying $\Sigma V = 0$ (↑+ve)

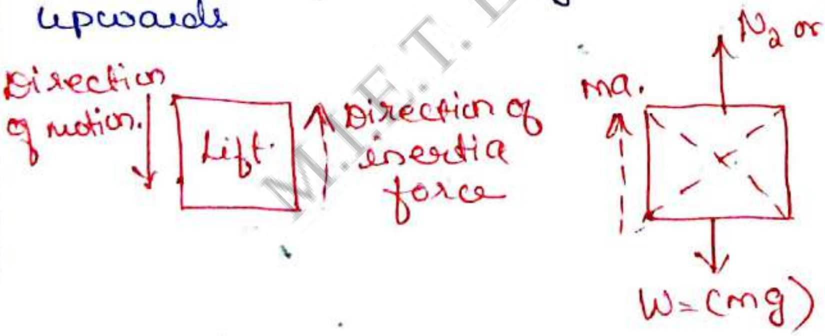
$N_1 - W - ma = 0$

$N_1 - W - \left(\frac{W}{g} \times a\right) = 0$

$$\boxed{N_1 = W\left(1 + \frac{a}{g}\right)}.$$

Case II : Lift is moving downwards:

when the lift is moving downwards, inertia force will act upwards



Applying $\Sigma V = 0$

$N_2 + ma - W = 0$

$N_2 + \frac{W}{g} a - W = 0$

$N_2 = W - \frac{W}{g} a$

$$\boxed{N_2 = W\left(1 - \frac{a}{g}\right)}$$

(2) A man weighing 600N gets into a lift. Calculate the force exerted by him on the floor of the lift, when it is
i) moving upwards with an acceleration of 3 m/s² and
ii) moving downwards with the same acceleration.

i) When the lift is moving upwards:

Force exerted by the man on the floor of lift, $N_1 = W\left(1+\frac{a}{g}\right)$

$= 600\left(1+\frac{3}{9.81}\right) = 783.48N$

ii) When the lift is moving downwards:

Force exerted by the man on the floor of lift $N_2 = W\left(1-\frac{a}{g}\right)$

$= 600\left(1-\frac{3}{9.81}\right) = 416.51N.$

(3) An elevator of weight (including wt. of man) 4.5 KN starts moving upwards with a const. acceleration and acquires a velocity of 1.8 m/s, after travelling a distance of 2m. Find the pull in the cable during accelerated motion. $\boxed{P-672}$

Given: $W = 4.5kN = 4500N.$

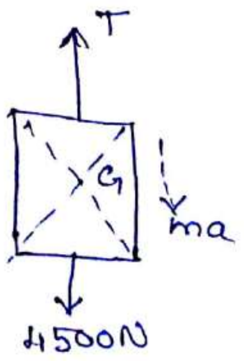
$u = 0$ (∵ starts moving from rest.

$v = 1.8 m/s.$, $s = 2m.$

using the equation, $v^2 = u^2 + 2as,$

$(1.8)^2 = 0 + (2 \times a \times 2)$

$a = \frac{1.8^2}{4} = 0.81 m/s^2$



Free Body diagram.

Lift is moving upwards. ∴ the inertia force is moving downwards.

Applying $\Sigma V = 0$ (↑+ve)

$T - 4500 - ma = 0 \Rightarrow T - 4500 - \left(\frac{4500}{9.81} \times 0.81\right) = 0$

$\boxed{T = 4871.5 N}$ Also can be cross checked using $N = W\left(1+\frac{a}{g}\right)$

Lecture No. 39

UNIT V – DYNAMICS OF PARTICLES

Topic(s) to be covered	Motion of connected bodies.

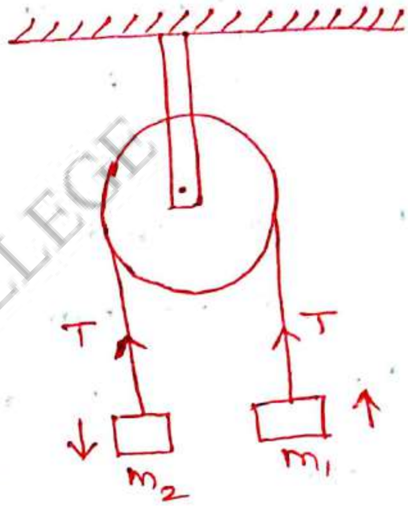
	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	apply knowledge on motion of connected bodies	L3

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

Motion of connected bodies:

Motion of two bodies connected by a string and passing over a smooth pulley.



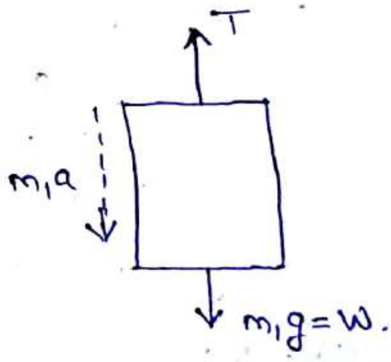
a = acceleration of bodies in m/s^2

T = Tension in the string in Newton.

consider mass m_1:

$m_1 < m_2$ hence it moves upwards.

The FBD of mass m_1, with inertia force $m_1 a$ is shown.



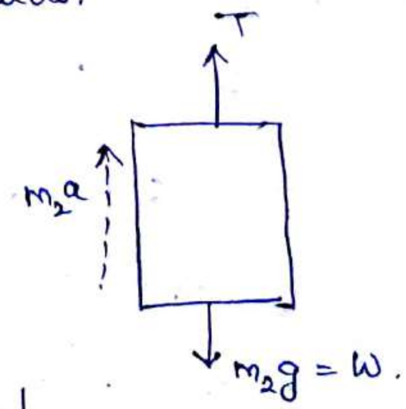
Applying $\Sigma V = 0$

$T_1 - m_1 g - m_1 a = 0$

$T - m_1 g = m_1 a$ ----- (1)

consider the mass m_2:

$m_2 > m_1$. Hence m_2 moves downwards.



Applying $\Sigma V = 0$

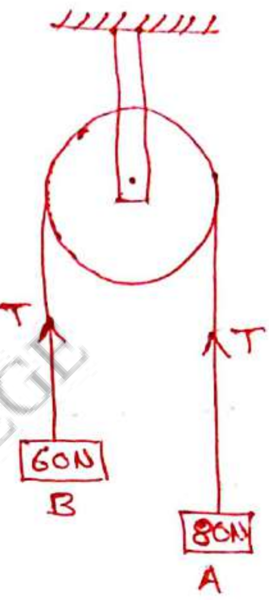
$T_1 + m_2 a - m_2 g = 0$

$m_2 a = m_2 g - T$ ----- (2)

Solving (1) and (2), we can get the unknown acceleration 'a' and tension in the string 'T'.

(4) Two blocks A and B of weight 80N and 60N are connected by a string, passing thro' a smooth pulley, as shown. Calculate the acceleration of the body and the tension in the string.

$W_A = 80N$; $W_B = 60N$.



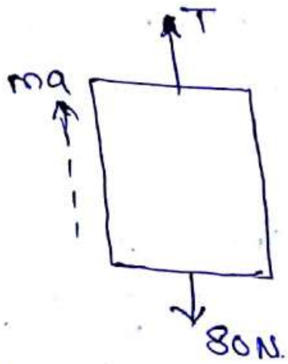
The string is passing thro' a smooth pulley and hence the tension on each side of the string will be equal.

wt. of block A (80N) > wt. of block B (60N)

∴ Block A moves downwards and block B moves upwards.

Considering block A (moving downwards):

The force and inertia force are shown.



Applying $\Sigma V = 0$ ($\uparrow +ve$).

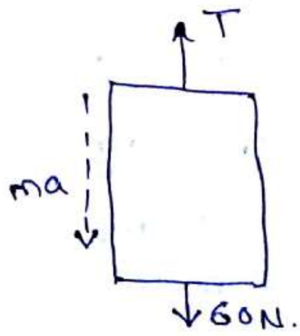
$T - 80 + ma = 0$

$T - 80 + \left(\frac{80}{9.81} a\right) = 0$

$T + 8.155a = 80$ ------ (1)

Considering block B (moving upwards).

Applying $\Sigma V = 0$ ($\uparrow +ve$)



$T - 60 - ma = 0$

$T - 60 - \left(\frac{60}{9.81} a\right) = 0$

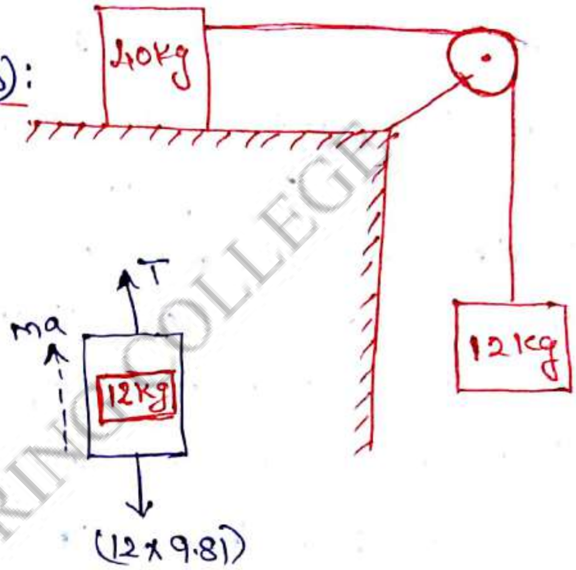
$T - 6.116a = 60$ ------ (2)

Solving equ. (1) and (2) we get 'a' and 'T'.

$\boxed{a = 1.401 m/s^2}$

$\boxed{T = 68.57N.}$

(5) A 40kg mass is dragged along the surface of a table by means of a cord which passes over a frictionless pulley at the edge of the table and is attached to a 12kg mass. If the co-eff. of friction between 40kg mass and the table is 0.15, determine the acceleration of the system and the tension in the cord.



Consider 12kg block (moving downwards):

Applying $\Sigma V = 0$ ($\uparrow$ +ve).

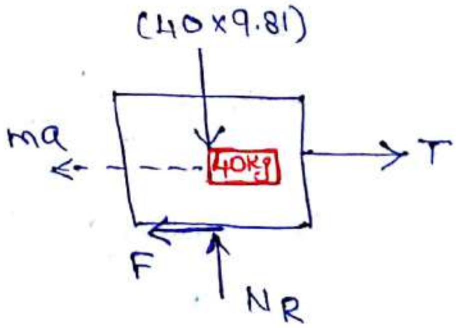
$$T + ma - (12 \times 9.81) = 0$$

$$T + 12a = 117.72 \quad \cdots\cdots (1)$$

FBD with inertia force.

Consider 40kg block:

(moving horizontally right side):



Applying $\Sigma V = 0$ ($\uparrow$ +ve).

$$N_R - (40 \times 9.81) = 0$$

$$\therefore N_R = 392.4 \text{ N}.$$

Applying $\Sigma H = 0$ ($\rightarrow$ +ve).

$$T - F - ma = 0$$

$$T - (\mu N_R) - ma = 0$$

$$T - (0.15 \times 392.4) - 40a = 0$$

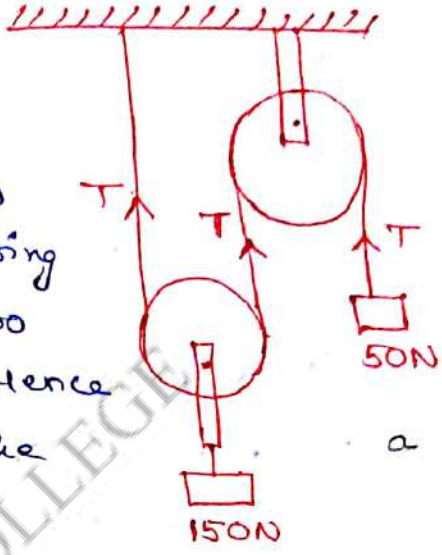
$$T - 40a = 58.86 \quad \cdots\cdots (2)$$

Solve (1) & (2), we get

$\boxed{a = 1.132 \text{ m/sec}^2}$ $\boxed{T = 104.13 \text{ N}.}$

(6) Two blocks of weight 150N and 50N are connected by a string and passing over a frictionless pulley as shown. Determine the acceleration of blocks A and B and the tension in the string.

Given:



wt. of blocks : 150N and 50N.

Here, acceleration of these two blocks will not be equal. because, the 50N block is supported by a single string But 150N block is supported by two strings. (i.e string on either side). Hence acceleration of block 50N is twice the acceleration of block 150N.

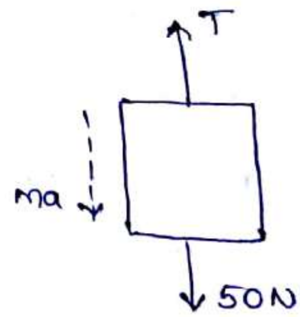
Let a = acceleration of block 50N wt.

150N block moves downwards and hence 50N upwards.

consider 50N block (moving upwards):

Applying $\Sigma V = 0$

$W = mg$
$m = \frac{W}{g}$



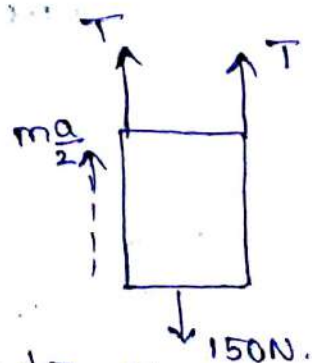
$$T - 50 - ma = 0$$

$$T - 50 - \left(\frac{50}{9.81}a\right) = 0$$

$$T - 5.09a = 50 \text{ ..... (1).}$$

consider 150N block (moving downwards):

Applying $\Sigma V = 0$; Note: $a = \frac{a}{2}$ and Tension = 2T.



$$2T - 150 + \frac{ma}{2} = 0$$

$$2T - 150 + \left(\frac{150}{9.81} \times \frac{a}{2}\right) = 0$$

$$2T + \left(15.29 \times \frac{a}{2}\right) = 150.$$

$$2T + 7.645a = 150 \text{ ..... (2)}$$

Solving equ. (1) and (2) $\Rightarrow$ $a = 2.805\ m/s^2$ and $T = 64.278N$.

$\therefore$ Acceleration of 50N block $(a) = 2.805\ m/s^2$
" " 150N block $\left(\frac{a}{2}\right) = 1.402\ m/s^2$.

Tension in the string $(T) = 64.28\ N$

P-211.

Lecture No.

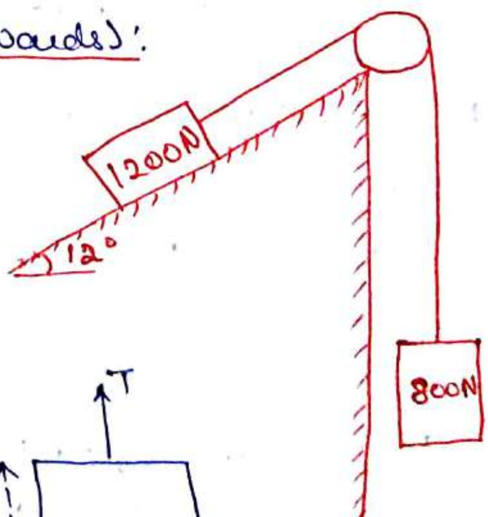
Topic(s) to be covered	

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

(7) A body weighing 1200N rests on a rough plane inclined at 12° to the horizontal. It is pulled up the plane by means of a light flexible rope running parallel to the plane and passing over a light frictionless pulley at the top of the plane as shown. The portion of the rope beyond the pulley hangs vertically down and carries a wt. of 800N at its end. If the co-eff. of friction for the plane and the body is 0.2, find (i) tension in the rope (ii) acceleration with which the body moves up the plane and (iii) the distance moved by the body in 3 seconds after starting from rest.



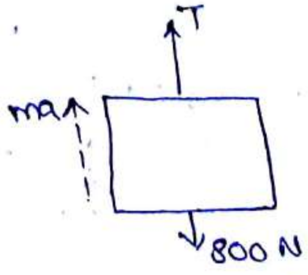
Consider 800N weight (moving downwards):

Applying $\Sigma V = 0$ ($\uparrow +ve$)

$$T + ma - 800 = 0$$

$$T + \left(\frac{800}{9.81}a\right) - 800 = 0$$

$$T + 81.549a = 800 \quad \cdots (1)$$



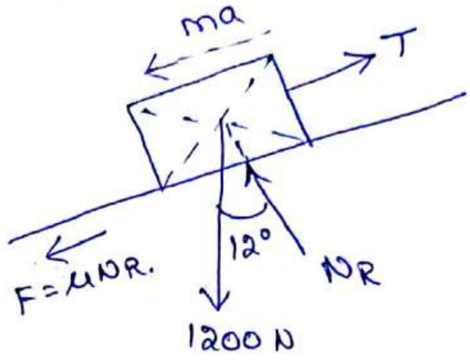
FBD of 800N block

Consider 1200N block on the inclined plane:

Resolving forces normal to the plane:

$N_R - 1200 \cos 12^\circ = 0$

$\boxed{N_R = 1173.78\,N}$



$\therefore F = \mu N_R$

$= 0.2 \times 1173.78$

$\boxed{F = 234.756\,N}$

Resolving forces along the plane:

$T - F - 1200 \sin 12^\circ - ma = 0$

$T - 234.756 - 1200 \sin 12^\circ - \left(\frac{1200}{9.81} a\right) = 0$

$T - 484.25 - 122.32a = 0$

$T = 122.32a + 484.25 \quad \ldots\ldots (2)$

Sub. equ. (2) in (1),

$\boxed{a = 1.548\,m/s^2}$ $\boxed{T = 673.76\,N}$

Distance moved in 3 seconds:

$t = 3$ sec.

$a = 1.548\,m/s^2$

$s = ?$

$u = 0$ ($\because$ starts from rest).

using the equation,

$s = ut + \frac{1}{2} at^2$

$= 0 + \left(\frac{1}{2} \times 1.548 \times 3^2\right)$

$\boxed{s = 6.966\,m}$

Topic(s) to be covered	Kinetics of particles - Impulse and momentum.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
LO1	understand the concept of impulse and momentum	L2.

Teaching Learning Material	Student Activity
Chalk and Talk	Learn and Solve

Lecture Notes

Kinetics of particles – Impulse and momentum.

Impulse-momentum method is based on integration of kinetic equations of motion w.r.t. time

Impulse of a force:

When a large force, acts for a short period of time, that force is called an impulsive force.

$I = \int_{t_1}^{t_2} F\,dt$ = Impulse of a force F acting over time from t_1 and t_2. If F remains const. during time interval (t_2-t_1) then $F(t_2-t_1)$ is called linear impulse or simply impulse.

$\boxed{\text{Linear impulse} = \text{Force} \times \text{time}}$ unit: Nsec.

Momentum:

is the quantity of motion possessed by a moving body.

Momentum = mass x velocity

$\boxed{M = mv}$ m in kg
v in m/s
M is linear momentum in kg/sec.

Impulse-momentum equation:

Impulse = Final momentum − Initial momentum.

Final momentum = Impulse + Initial momentum.

Impulse of a force acting on a particle is equal to the change in the linear momentum of the particle.

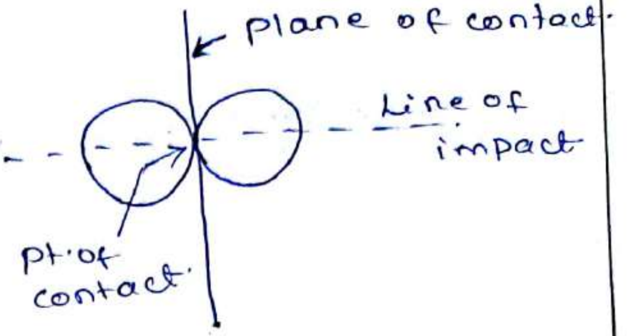
$\boxed{\text{Impulse} = m(v-u)}$

$= \frac{W}{g}(v-u).$

Impact of elastic bodies

General terms:

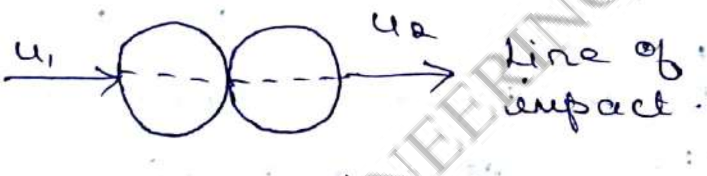
1. Plane of contact.
2. Line of impact.
3. Period of deformation.
4. Period of restitution.



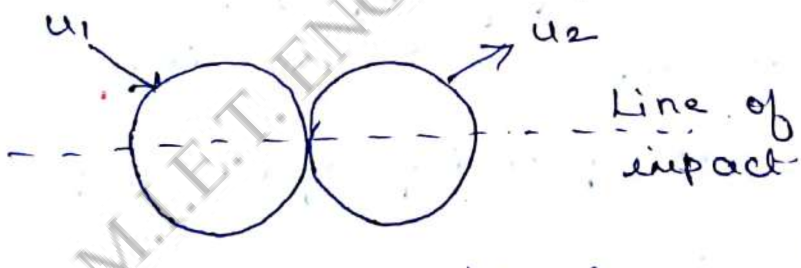
Types of impact:

1. Direct impact.
2. oblique impact.

In direct impact, the velocities of the two colliding bodies before collision are collinear with the line of impact.



Direct impact.



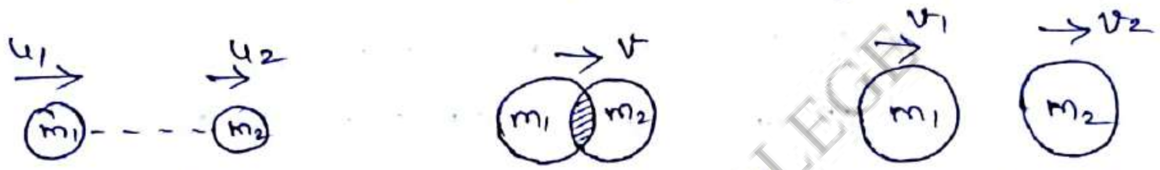
oblique impact.

In oblique impact, the velocities of the colliding bodies are not directed along the line of impact and the line of impact passes thro' the mass centre of the colliding bodies.

Period of Restitution and co-eff. of restitution:

The time elapsed from the instant of initial contact to the max. deformation is known as period of deformation and the time elapsed from the instant of max. deformation to the instant of just seperation of particles is known as period of restitution.

$$\text{co-eff. of restitution} = \frac{\text{Impulse during restitution}}{\text{Impulse during deformation}}$$



a) Before impact. b) max. deformation during impact. c) after impact.

(i) Law of conservation of momentum:

When two elastic bodies collide each other, the momentum gained by one body must be equal to the momentum lost by the other body.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

ii) Newton's Law of collision of elastic bodies:

It states that "for two colliding bodies, their relative velocity of seperation bears a const. ratio to their relative velocity of approach."

$$(v_2 - v_1) = e(u_1 - u_2).$$

① A sphere of mass 1kg moving with a velocity 2 m/s impinges directly on a sphere of mass 2kg at rest. If the first sphere comes to rest after the impact, find the velocity of the second sphere and the co-eff. of restitution.

Given: $m_1 = 1kg$ $u_1 = 2 m/s$ $v_1 = 0$
$m_2 = 2 kg.$ $u_2 = 0$ $v_2 = ?$ $e = ?$

From law of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(1 \times 2) + (2 \times 0) = (1 \times 0) + (2 \times v_2)$$

$$\therefore v_2 = 1 m/s$$

From Newton's law of collision of bodies,

$$v_2 - v_1 = e\,(u_1 - u_2)$$

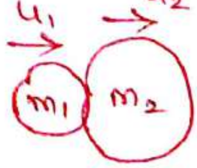
$$(1 - 0) = e\,(2 - 0)$$

$$\therefore e = \frac{1}{2} = 0.5$$

$\therefore$ the velocity of the second sphere after impact is 1m/s and the co-eff. of restitution is 0.5.

Assignment / tutorial:

A ball strikes centrally on another ball of mass twice the mass of 1st ball but moving with a velocity $\frac{1}{7}$ of the velocity of first ball and in the same direction, show that the first ball comes to rest after impact. The co-eff. of restitution bet. them is 3/4.

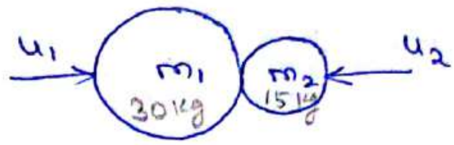


② Two bodies one of mass 30 kg, moves with a velocity of 9 m/s strikes on another body of mass 15 kg, moving in the opp. direction with a velocity of 9 m/s centrally. Find the velocity of each body after impact, if the co-eff. of restitution is 0.8.

Given:

$m_1 = 30 kg$ $\quad u_1 = 9 m/s$

$m_2 = 15 kg$ $\quad u_2 = -9 m/s.$



(-ve sign due to opp. direction)

$e = 0.8$, $v_1 = ?$, $v_2 = ?$

From the law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(30 \times 9) + (15 \times -9) = 30 v_1 + 15 v_2.$$

$$30 v_1 + 15 v_2 = 135 \quad \ldots ①$$

From the newton's law of collision.

$$(v_2 - v_1) = e\,(u_1 - u_2)$$

$$(v_2 - v_1) = 0.8\,(9 - (-9))$$

$$v_2 - v_1 = 0.8(18) = 14.4 \quad \ldots ②$$

Solving ① & ②, we get,

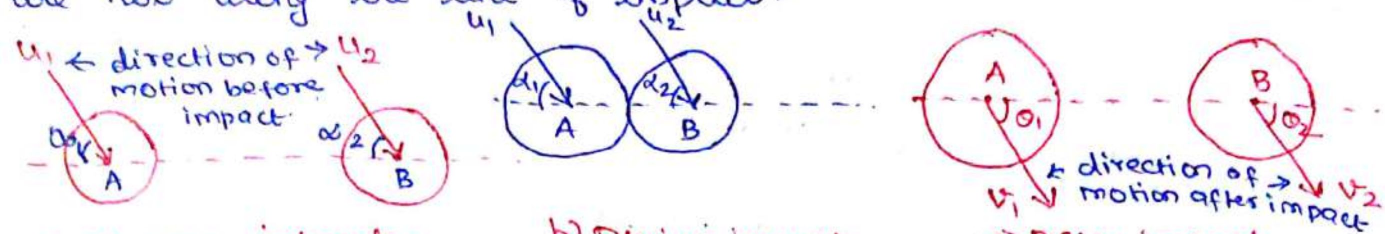
$v_1 = -1.8 m/s$ (-ve sign indicates, direction is reversed).

$v_2 = 13 m/s.$

$\therefore$ After impact, the 1st body will move with a velocity of 1.8 m/s (←) and the 2nd body will move with a velocity of 13 m/s (→).

oblique central impact.

In oblique central impact, the mass centre of the bodies lies on the line of impact, but the bodies are not moving along the line of impact, i.e. the initial velocities of the bodies are not along the line of impact.



a) Before impact. b) During impact c) After impact.

Note: Due to collision, the vertical components of velocities are not affected.

horz. components of velocity: $u_1 \cos\alpha_1$, $u_2 \cos\alpha_2$; $v_1 \cos\theta_1$, $v_2 \cos\theta_2$.

vertical component of velocity: $u_1 \sin\alpha_1$, $u_2 \sin\alpha_2$; $v_1 \sin\theta_1$, $v_2 \sin\theta_2$.

Due to collision, the vertical components of velocities are not affected

$$\therefore \boxed{\begin{aligned} u_1 \sin\alpha_1 &= v_1 \sin\theta_1 \\ u_2 \sin\alpha_2 &= v_2 \sin\theta_2 \end{aligned}}$$

① Law of conservation of momentum:

$$m_1 u_1 \cos\alpha_1 + m_2 u_2 \cos\alpha_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2.$$

② Newton's law of collision:

Relative velocity of seperation $= (v_2 \cos\theta_2 - v_1 \cos\theta_1)$.

Relative velocity of approach: $u_1 \cos\alpha_1 - u_2 \cos\alpha_2$.

$$\therefore v_2 \cos\theta_2 - v_1 \cos\theta_1 = e(u_1 \cos\alpha_1 - u_2 \cos\alpha_2).$$

e = co-eff. of restitution.

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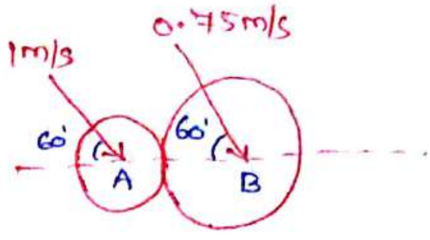
(1) A ball of mass 500 gms, moving with a velocity of 1 m/s impinges on a ball of mass 1 kg, moving with a velocity of 0.75 m/s. At the time of impact, the velocities of the balls are parallel and inclined at 60° to the line joining their centres. Determine the velocities and directions of the balls after impact. Take e = 0.6.

$m_1 = 0.5\,kg$; $u_1 = 1\,m/s$; $\alpha_1 = 60°$

$m_2 = 1\,kg$; $u_2 = 0.75\,m/s$; $\alpha_2 = 60°$



Calculate v_1, v_2, θ_1 and θ_2.

The vertical components of velocities before and after impact are the same.

ie
$$\boxed{\begin{aligned} u_1 \sin\alpha_1 &= v_1 \sin\theta_1 \\ u_2 \sin\alpha_2 &= v_2 \sin\theta_2 \end{aligned}}$$

$\Rightarrow u_1 \sin\alpha_1 = 1 \times \sin 60° = 0.866 = v_1 \sin\theta_1$ .... (1)

$\Rightarrow u_2 \sin\alpha_2 = 0.75 \times \sin 60° = 0.649 = v_2 \sin\theta_2$ .... (2)

~~$v_1 \sin\theta_1$~~

Applying law of conservation of momentum along the line of impact:

$$m_1 u_1 \cos\alpha_1 + m_2 u_2 \cos\alpha_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2.$$

$$(0.5 \times 1 \times \cos 60°) + (1 \times 0.75 \times \cos 60°) = (0.5 v_1 \cos\theta_1) + (1 \times v_2 \cos\theta_2)$$

$$0.5 v_1 \cos\theta_1 + v_2 \cos\theta_2 = 0.625 \quad \text{.... (3)}$$

Applying Newton's law of collision along the line of impact.

$$(v_2 \cos\theta_2 - v_1 \cos\theta_1) = e\,(u_1 \cos\alpha_1 - u_2 \cos\alpha_2)$$

$$= 0.6\,(1 \times \cos 60° - 0.75 \cos 60°)$$

$$= 0.075.$$

$$v_2 \cos\theta_2 - v_1 \cos\theta_1 = 0.075 \quad \text{.... (4)}$$

Solving equations (3) and (4),

$$v_1 \cos\theta_1 = 0.367 \quad \text{.... (5)}$$

$\frac{\text{equ. (1)}}{\text{equ. (5)}} \Rightarrow \frac{v_1 \sin\theta_1}{v_1 \cos\theta_1} = \frac{0.8666}{0.367}$ (or) $\tan\theta_1 = 2.359$.

$\therefore \boxed{\theta_1 = 67°}$

sub. θ_1 in equ. (5)

$$v_1 \cos\theta_1 = 0.367.$$

$$\therefore \boxed{v_1 = 0.939 \text{ m/s}}$$

from equ. (4),

$$v_2 \cos\theta_2 - v_1 \cos\theta_1 = 0.075.$$

$$v_2 \cos\theta_2 = 0.075 + 0.367.$$

$$v_2 \cos\theta_2 = 0.442 \quad \ldots\ldots (6)$$

$$\frac{\text{equ. (2)}}{\text{equ. (6)}} \Rightarrow \frac{v_2 \sin\theta_2}{v_2 \cos\theta_2} = \frac{0.649}{0.442}.$$

$$\tan\theta_2 = 1.468.$$

$$\therefore \theta_2 = \tan^{-1}(1.468) = 55.74°.$$

Sub. θ_2 in (6),

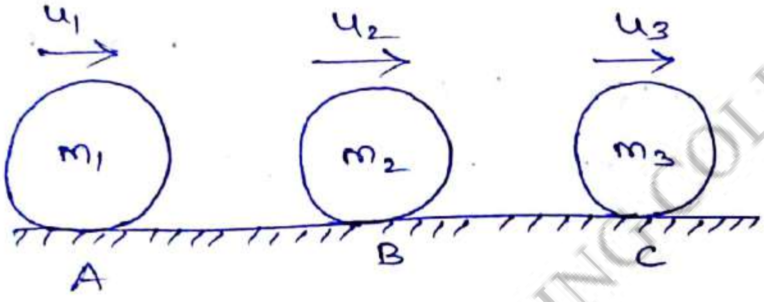
$$v_2 \cos\theta_2 = 0.442$$

$$v_2 \cos 55.74° = 0.442.$$

$$\therefore \boxed{v_2 = 0.785 \text{ m/s.}}$$

Three spherical balls of wt. 20N, 60N and 120N are moving in the same direction with velocities 12 m/s, 4 m/s and 2 m/s respectively. If the ball of the weight 20N impinges with the ball of weight 60N which in turn impinges with the ball of 120N, prove that the balls of weights 20N and 60N will be brought to rest after the impact. Assume the balls to be perfectly smooth.

Let the balls be A, B and C moving in the same direction as shown.



Given: $W_1 = 20N$; $m_1 = \frac{20}{9.81} = 2.04 kg$; $u_1 = 12$ m/s

$W_2 = 60N$; $m_2 = \frac{60}{9.81} = 6.12 kg$; $u_2 = 4$ m/s

$W_3 = 120N$; $m_3 = \frac{120}{9.81} = 12.24 kg$; $u_3 = 2$ m/s.

First, consider the collision of the balls A and B:

From the law of conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(2.04 \times 12) + (6.12 \times 4) = (2.04 \times v_1) + (6.12 \times v_2)$$

$$2.04 v_1 + 6.12 v_2 = 48.96.$$

$$3v_2 + v_1 = 24 \quad \text{......} (1)$$

Applying Newton's law of collision,

$$(v_2 - v_1) = e\,(u_1 - u_2)$$
$$= 1(12-4)$$

$$\therefore v_2 - v_1 = 8 \quad \text{....} (2)$$

Solving equations ① and ②, $v_1 = 0$ and $v_2 = 8$ m/s.
Here $v_1 = 0$, i.e. after the impact, the first ball comes to rest, but the second ball is moving with a velocity of 8 m/s.

Now, consider the collision of balls B and C:

$m_2 = 6.12$ kg ; $u_2 = 8$ m/s [v_2 in the collision of A & B]
$m_3 = 12.24$ kg ; $u_3 = 2$ m/s.

From the law of conservation of momentum,

$$m_2 u_2 + m_3 u_3 = m_2 v_2 + m_3 v_3.$$ [here v_2 is not same as in the collision in A and B].

$$(6.12 \times 8) + (12.24 \times 2) = 6.12 v_2 + 12.24 v_3.$$

$$6.12 v_2 + 12.24 v_3 = 73.44$$

$$v_2 + 2 v_3 = 12 \quad \text{.....} ③$$

From Newton's law of collision:

$$(v_3 - v_2) = e(u_2 - u_3)$$

$$= 1(8-2)$$

$$v_3 - v_2 = 6 \quad \text{.....} ④$$

Solving equ. ③ and ④,

$v_2 = 0$, $v_3 = 6$ m/s.

$v_2 = 0$ i.e. after the impact of ball B with Ball C, ball B comes to rest.